FINANCE 700 Dr. J. Stuart Wood

ATTENDANCE POLICY:

Students are expected to attend all classes, arrive on time, not leave early, have cell phones and pagers disabled, behave professionally, answer questions posed by the instructor, and participate in class discussion. "Preparation precedes progress." Class attendance is absolutely required. Roll will be taken daily. If you must miss class, telephone me prior to that class at (504) 495-6443 and tell me why you will miss class; leave a message if I do not answer. At the next class, turn in a written note explaining your absence. It is better to arrive late to class than to miss the entire class. If you expect to be late, telephone me before you are late at (504) 495-6443. If you do arrive late, do not wait in the hall, but come immediately into class, and we will welcome you enthusiastically.

FINANCE 700 ACADEMIC INTEGRITY CONTRACT:

I _________________________________________________,

understand and agree to the following:

Each case solution you turn in must be ENTIRELY your own work. You may discuss the case with others, but you may not write or type on a keyboard while discussing. You must personally and individually create and prepare all of the spreadsheets required for the case. The cover sheet of each report must contain your signed statement that the work is entirely your own and that you neither gave assistance nor received assistance in performing the analysis, creating the spreadsheets, or in writing the text. You may discuss finance with anyone and ask analytical questions regarding the method of analysis, but you may not write down what you are told at that time: you may repeat it verbally as many times as you like, but all of your writing must be done in isolation. You may get all the assistance you wish regarding the composition of the report from the Writing Laboratory or from me. You may get all the assistance you wish from me personally. You may not work with any other person on a case solution or the creation of spreadsheets. You may discuss the specific case materials with another person. You may discuss the finance concepts, analysis, methods, anything at all about finance and not specifically the case, with anyone else.

All work in this course is strictly individual. You must do yourself all work which you turn in for the course. All work is take-home and is open-book. You may discuss concepts and principles with each other and discuss how to do the work, and you may ask me any question at any time; but you may not e-mail spreadsheets or text to each other, nor may you copy spreadsheet equations or text from each other, nor may you provide text or spreadsheets to another. If you take notes during a discussion with another person, attach a copy of those notes to your paper. Ask me any questions you may have. Ask me explicitly whether or not I approve or disapprove any specific action which might possibly be against my rules, prior to the commission of that action. The penalty for compromise of your academic integrity by receiving live written help from another student or giving written help to another student, or by working together to build spreadsheets, is disenrollment from the course with a grade of "F". See the Loyola University New Orleans Graduate Bulletin.

J. Stuart Wood

I accept and agree to this policy and I promise to abide by it in full.

Name: __________________________________________
Signature:________________________________________
Date: __________________________________________
Witness #1: _______________________________________
Witness #2: _______________________________________
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All work in this course is strictly individual. You must do yourself all work which you turn in for the course. All work is take-home and is open-book. You may discuss concepts and principles with each other and discuss how to do the work, and you may ask me any question at any time; but you may not e-mail spreadsheets or text to each other, nor may you copy spreadsheet equations or text from each other, nor may you provide text or spreadsheets to another. If you take notes during a discussion with another person, attach a copy of those notes to your paper. Ask me any questions you may have. Ask me explicitly whether or not I approve or disapprove any specific action which might possibly be against my rules, prior to the commission of that action. The penalty for compromise of your academic integrity by receiving live written help from another student or giving written help to another student, or by working together to build spreadsheets, is disenrollment from the course with a grade of "F". See the Loyola University New Orleans Graduate Bulletin.

J. Stuart Wood

I accept and agree to this policy and I promise to abide by it in full.

Name: __________________________________________

Signature:________________________________________

Date: __________________________________________

Witness #1: _______________________________________

Witness #2: _______________________________________
LOYOLA UNIVERSITY COLLEGE OF BUSINESS ADMINISTRATION
FINANCE  B 700-051--- ADVANCED FINANCIAL MANAGEMENT
Pre-requisite: Finance 601, Accounting 601
First Six-Week Session, SUMMER, 2008, Dr. J. Stuart Wood

A normal semester lasts 13 weeks; there are here only 5 weeks and 10 classes: each class covers more than one week’s material.

Tuesday and Thursday evenings, 6:00 PM – 9:20 PM, Miller 204
May 27, 2008 through July 1, 2008; Final Grades Posted on Wednesday, July 9, 2008
My Office: Miller Hall 316: Mondays, Tuesdays, Wednesdays, and Thursdays 5:00 PM – 6:00 PM
Cell Phone: (504) 495-6443; Home telephone: (504) 866-7200; Fax: (504) 834-9358

I DO NOT USE E-MAIL. DO NOT SEND ME ANY E-MAIL!!
Please telephone me anytime you have a question or wish to talk.

TEXTBOOKS:
   There is a very useful Compact Disc with spreadsheet programs with the book. Note Appendix B pp. 984-990 “Answers to Quizzes,” Appendix C pp. 991-1002 “Selected Equations and Data,” and “Glossary” pp. 1003-1023. We will cover Chapters 1-5, 7, 9-13, 15-17.

FINANCE 700 CATALOG DESCRIPTION:
Entrepreneurial Financial Management is the Imagining and Selection of the Course of Action Which Most Increases the Present Wealth of Equity Holders of the Firm:

\[ PWE_0 = NCDE_0 + VE_0; \quad \Delta PWE_0 = NPV_0 \]

Valuation is the crux of finance. Valuation is subjective and proceeds from forecasted cash flows and Terminal Value meeting the forecast assumptions within the identified course of action, all discounted to the present at the risk-adjusted cost of capital. No market process has a determinate outcome. Successful entrepreneurship is imagining and then choosing the future course of action which most increases the value of equity. A decision changes the future course of action from the current plan to a new and better plan. Valuing the equity of the firm under the present plan, and changes in that value under newly-designed alternative plans, is the goal of this course: first, by forecasting the future pro-forma financial statements through the Terminus ("horizon date") T under the forecast assumptions inherent in the identified course of action; second, by forecasting the free cash flows through the Terminus T; third, by forecasting the Terminal Value at the Terminus T; and fourth, by discounting the stream of future forecasted cash flows and Terminal Value at the risk-adjusted cost of equity to the present value of equity under the forecast assumptions for the identified course of action. We shall explore correct financial decision-making in a variety of situations: the correct decision is to choose the course of action which increases most the value of equity of the firm. For each decision, all of the alternative courses of action identified must be forecasted so the best one can be selected. The present value of equity of the proposed future course of action is compared with the present value of equity of the existing situation, and the increase in value of equity if the proposed course of action is implemented, is determined. The correct decision in any situation is to choose the course of future action which most increases the value of equity of the firm, after explicitly forecasting the future cash flows which will result from each possible course of action identified at the decision moment.
Management has three broad areas of decisions to make regarding financial affairs, and also
values the entire firm:

**Capital investment:** Selection of particular capital assets to increase PWE

\[
PWE_0 = NCDE_0 + VE_0 \quad \Delta PWE_0 = NPV_0.
\]

\[
NPV = -IVS_0 + \sum_{t=1}^{T} \frac{FCF_t^*}{(1+kf^*)^t} + TV_T / (1+kf^*)^T > 0);
\]

\[
DOL = \Delta %EBIT / \Delta %SALES; \quad g^*; \quad ke = R_f + \beta (E[R_M] - R_f);
\]

\[
ke = (d_1/P_0) + g\quad ; \quad ke = kd + 0.05; \quad g^* = PRAT^*
\]

**Capital structure:** Choosing the capital structure Θ to minimize the cost of capital kf^*

\[
\Theta = VF/VF; \quad Hamada \ equation: \quad \Delta \Theta \rightarrow \Delta \beta; \quad \Delta \Theta \rightarrow \Delta g^*; \quad \Delta \Theta \rightarrow \Delta g^*;
\]

\[
kf^* = \Theta kd^* + (1-\Theta) ke; \quad DFL = \Delta NIAT / \Delta %EBIT; \quad VF = VD + VE
\]

**Capital acquisition and disbursement:** How to raise and distribute capital, interest, and dividends.

\[
NDC, NEC, CDD, CDE; \quad NCDE = CDE - NEC;
\]

\[
NCDD = CDD - NEC; \quad I_t = VD_t-1 \times kd; \quad p, b, \rightarrow \Delta g^*; \rightarrow \Delta g^*.
\]

**Valuation of the Equity of the Firm:** What is the ownership position worth? VE = ?

\[
VE_0 = \sum_{t=1}^{T} \frac{LFCFE_t}{(1+ke)^t} = \sum_{t=1}^{T} \frac{LFCFE_t}{(1+ke)^t} + TV_T / (1+ke)^T
\]

**Valuation of the Firm:** What are all the assets worth? VF = VE + VF = ?

\[
VF_0 = \sum_{t=1}^{T} \frac{UFCFF_t}{(1+kf^*)^t} = \sum_{t=1}^{T} \frac{UFCFF_t}{(1+kf^*)^t} + TV_T / (1+kf^*)^T
\]

Decisions are made by identifying, imagining, and creating superior future states of the firm,
compared with the present anticipation, and then selecting the best of the possible forecasted futures.

“The outcomes of market processes depend on what happens at their various stages and on the order in
which events happen. This means in particular that antecedents will influence subsequent events in so far as acting men attribute significance to them and that therefore the order in which events happen
matters…”

1. **Financial and Capital Market Processes Within Which the Firm Operates:**
   May 27: Chapters 1, 4, 5, Gordon Growth Model
   May 29: Chapter 2 Portfolio Theory; Chapter 3 Capital Asset Pricing Model
   June 3: Sustainable Growth Model, Financial Cash Flows Chapter 7

2. **Making Internal Corporate Decisions Based Upon External Market Processes and Data:**
   June 5: Forecasting Financial Statements—BD Chap. 9; SBW Chap. 10
   June 10: Free Cash Flows to Equity and to All Capital Suppliers--SBW Chap. 12
   June 12: The Cost of Capital of the Firm—BD Chap. 10, SBW Chap. 11
   June 17: Corporate Valuation—BD Chap. 11.
   June 19: Capital Budgeting—BD Chap. 12, 13.
   June 24: Capital Structure Optimization—BD Chaps. 15-16.
   June 26: Capital Acquisition and Disbursement Decisions—BD Chap. 17.
   July 1: RMAG Simulation Valuation Case is Due.

**FINANCE 700 ASSIGNMENTS**

Interest and Discounting Practice Problems: Due May 29: Chap 1 p. 28-29 (1,2);
Chap 4 pp. 145-146 (2, 3, 4), pp. 146-147 (1, 2, 3, 13).
Due June 3: Chap. 5 pp. 182-184 (1, 2, 3, 5, 6, 7, 9, 10, 12, 14).
Risk and Return Practice Problems: Due June 5: Chap 2 p. 65 (3, 4),
p. 66-69 (1, 2, 3, 4, 5, 12,14); Chap 3 pp. 107 (2), p. 109 (3, 4).
Case 1: Due June 10—Forecasting Wal-Mart--SBW Prob. 10.16, a, b, c only; pp.787-794 (275-278)
Case 2: Due June 17—Gap & Limited (Handed out) Stickney (V) Problem 10.7 a, b, c, d
Case 3: Due June 24—Massachusetts Stove Co. SBW Case 10.2 b, c, d, pp. 802-811
Case 4: Due July 1—Simulation Valuation (Handed out) of Rocky Mountain Advanced Genome

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DISABILITY STATEMENT:
A student with a disability that qualifies for accommodations should contact Sarah Mead Smith, Director of Disability Services at 865-2990 (Academic Resource Center, Room 405, Monroe Hall). A student wishing to receive test accommodations (e.g., extended test time) should provide the instructor with an official Accommodation Form from Disability Services in advance of the scheduled test date.

“The market is a process of creation, discovery and adjustment.”
"Entrepreneurship is the alert creation and pursuit of previously-unsuspected opportunities."
--Israel Kirzner

"The future is unknowable but not unimaginable. No market process has a determinate outcome. We live in a world of unexpected surprise."
--Ludwig Lachmann

"With wings I have won for myself, in fervent love I shall soar to the Light which no eye has seen....What has battered you, my Soul, will bear you to God."
--Gustav Mahler
REQUIRED STANDARDS AND FORMAT FOR WRITTEN CASE SUBMISSIONS

Reference: Strunk and White, The Elements of Style
You should have your own copy of Strunk and White. The Bookstore sells this book. Buy it and read it repeatedly. Learn to write elegantly and coherently.

Each case solution you turn in must be ENTIRELY your own work. You may discuss the case with others, but you may not write or type on a keyboard while discussing. You must personally and individually create and prepare all of the spreadsheets required for the case. The cover sheet of each report must contain your signed statement that the work is entirely your own and that you neither gave assistance nor received assistance in performing the analysis, creating the spreadsheets, or in writing the text. You may discuss finance with anyone and ask analytical questions regarding the method of analysis, but you may not write down what you are told at that time; you may repeat it verbally as many times as you like, but all of your writing must be done in isolation. You may get all the assistance you wish regarding the composition of the report from the Writing Laboratory or from me. You may get all the assistance you wish from me personally. You may not work with any other person on a case solution or the creation of spreadsheets. You may discuss the specific case materials with another person. You may discuss the finance concepts, analysis, methods, anything at all about finance and not specifically the case, with anyone else.

You may see me in my office at Loyola (MI 306) or telephone me at 866-7200 or 495-6443 to ask any question or receive assistance from me regarding any aspect of preparing your case solution. Please do not call after 9:00 PM.

Case solutions are due at the start of class on assigned days. Late submissions will receive a grade of zero. If you fail to attend class on a day scheduled for your presentation, you will receive two zeroes.

Provide a cover sheet showing your name, the name and number of the course, the title and number of the case, and the due date. Sign a statement of independent work on the cover page. Put a header on each page of the case showing your name, the date, the case title, and the page number.

Case solutions must be typed or printed, on one side of each page. Margins must be 1.5" at the top and 0.75" on bottom and sides. The header should be three lines, beginning 0.5" from the top, showing your name, the case title, the date, and the page number. Do not use type larger than ten (10) characters per inch. Pages must be numbered. Papers must be stapled together. Writing in the "discussion and analysis section" may be double-spaced; the cover letter is single-spaced. The executive summary can be single-spaced if necessary to fit within the two-page limit. Use titles for sections, where appropriate.

Use standard English prose style and write professionally, using third person constructions generally, except where first or second person is appropriate: you are writing about the company in question, not about yourself. You may use first person to give your opinion regarding the company or a forecast. Prefer active voice over passive voice. In the United States, we use singular pronouns to refer to business firms. Be sure that the number and gender of your nouns and pronouns agree with the verbs you use. Write in complete sentences: fragments or run-on sentences will result in a heavy penalty (one letter grade) for each fragment or run-on sentence. If you are not a native English-speaker, you must still write correctly in standard English. Although it will be painful, I will give grades of F on the written case solutions if they are very badly written. I will not relent in insisting that you write absolutely correctly, and I will grade off heavily for poor writing.

FORMAT: Each case solution must contain the following sections:
0) Cover sheet with your name, course identification, and signed independence statement;
1) Cover Letter no longer than one page, stating the problem, stating what you did, giving the answer or recommendation, and stating how many pages of what types are attached;
2) Executive Summary no longer than 2 pages;
3) Analysis and Discussion at least five (5) full pages long, pages numbered separately, (Note that 4.75 pages is fewer than five pages);
4) Quantitative Appendix as long as necessary containing equations, tables, graphs, etc.

Structure and Content of Case Solutions:
Cases shall be typed or computer-printed, on one side of each page, with ordinary side and bottom margins of 0.75", a top margin of 1.5" to accommodate the running head, and with type not fewer than 10 characters per inch. In addition to the Transmittal Letter (one page) and Executive Summary (two pages), the case solution shall contain not fewer than five full pages of text: following the text will be the numerical tables and graphs. The tables and graphs should be collected into an Appendix to the formal report. Be sure that you give the answer or solution to the case in the cover letter.

Case reports which do not meet the minimum standards of length will receive an automatic grade reduction of two letter grades.
CONTENTS OF CASE REPORT

General: Do not describe the company to the reader: I have read the case, and the reader is familiar with the history of the company and its products. In your role as consultant or employee, you are writing a report to the president or other officer of the firm, to a banker who knows the firm well, or somebody else connected with the firm. Therefore, you do not need to describe the history of the firm or its current business, unless some element of this is the key to a particular problem you have been asked to address. Do not waste valuable space describing the company's business, the company's locations and assets, the company's methods of production, or its history.

1. Transmittal Letter--1 page only, giving the answer to the problem.
A formal business letter from you, with your capacity identified (consultant, loan officer of bank, manager or officer of firm, potential investor, etc.), to the person for whom you are performing the analysis and making recommendations--the decision-maker (bank officer, chairman of loan review committee, board of directors, manager, vice-president of finance, owner, potential stockholder, potential bondholder, etc.). This letter is in the context of the case, using the appropriate names. The letter must have your letterhead and the inside address of the recipient. Do not use memorandum form. This is a formal business letter. You must in this letter give the answer to the problem and make the recommendation for action.

In the cover letter, in one or two paragraphs:

a.) state what you were asked to do.
b.) tell your client that you have completed the requested analysis.
c.) state the problem.
d.) give your recommendation or solution, and
e.) give the major reason for your recommendation.
f.) Identify what you are delivering and
g.) state how many pages it has, what exhibits, tables, etc.

Note that the essence of the solution to the case must be given in the cover letter: GIVE THE ANSWER.
Say what you have decided, what the value is, whether the loan is granted or not, etc. Give the reason.

I mean that the cover letter should give the reader the statement of the problem and its solution. If nothing else can be read, the reader will know what was wrong with the firm and how to solve it.

Your cover letter must explicitly give the decision you have made. Your cover letter must give the answer to the problem.

2. Executive Summary—Exactly 2 pages or fewer.
DO NOT MERELY COPY PORTIONS OF YOUR ANALYSIS.
SUMMARIZE THE SITUATION, YOUR ANALYSIS, AND YOUR CONCLUSIONS.

a. Problem Statement--What problem does the firm face? Summary of the issues and problems you addressed.
b. Identification of Problem--How do you know this problem exists?
c. Alternative Solutions--the major points of analysis. What alternatives are available to solve the problem or overcome the difficulty?
d. Analysis of entrepreneurship, profitability, competitive position.
e. Results, Conclusions, Recommendations--What single course of action should be taken? What decision did you make? Why?

Do not merely copy a few paragraphs from your Analysis section for the Executive Summary; rather, summarize the problem and your analysis and solution.

3. Analysis and Discussion—5 or more pages.
Complete analysis and discussion of the reasons why you have performed the analysis you did, what you did, and what your results, conclusions, and recommendations are.

a. History and development of problem; description of the problem(s); categorization of the type(s) of problem(s); causes of problem.
b. Manifestation of problem--illustrated with ratios, cash flows, financial statement histories. Refer to Appendix as necessary.

c. Results if no action is taken; advantages and disadvantages of doing "nothing" (itself a decision); illustrated with ratios, cash flows, financial statement histories in the Appendix.

d. Alternative actions available or methods of solution, and analysis of likely outcomes of each action; advantages and disadvantages of each alternative action; illustrated with ratios, cash flows, financial statement projections (pro-forma statements and FCF projections) in the Appendix.

e. Entrepreneurial evaluation and analysis of issues of rivalrous competition and profitability.

f. Selection of the best course of action from the alternative available courses of action. What criteria do you use to identify the best course of action? Why do you choose this action? Provide a complete analysis and discussion of the reasons why you have performed the analysis you did, what you did, and what your results, conclusions, and recommendations are. Discuss the problems faced by the firm, the appropriate categorization of these problems, alternative methods of solution which could be pursued, the best solution method in your judgment, and a complete step-by-step description of all the analysis and computations you performed. You may report the results of calculations in this section, and show the equations and terms used, where they are appropriate.

Do not put spreadsheets and tables in this section; spreadsheets and tables are in the Appendix. But in this section, you refer to the information contained in the spreadsheets and tables, explain to the reader how to read each spreadsheet and table, and discuss the meanings of the numbers contained in the spreadsheets and tables. (For example: "Table 1 shows financial ratios for Atlantic-Southeast Airlines. The first ratio shown is the current ratio. We see that the current ratio has declined over the past four years from 2.7 to 0.9, indicating a severe shortage of current assets relative to current liabilities. The industry average is 1.6, so ASA is considerably less liquid than its competitors, and will have greater difficulties remaining solvent in the event of a decline in passenger volume or a fare war. This lamentable condition has come about because of the lag in collecting overdue accounts by ASA and their insistence on paying all suppliers timely. This pattern of lagging receipts and prompt payments has produced a cash shortage reflected in the current ratio. Such deficiencies in cash receipts must harm the profitability compared with competitors, and reduce the cash flow available to the equity holders and thereby depress the price of the stock."

4. APPENDIX: Financial Statements, Spreadsheets, Tables, Ratio Computations, Graphs--as many pages as necessary

Financial Cash Flow Statements supporting your decision
Funds Flow (Sources and Uses) Statements
Pro-forma financial statements or cash-flow statements for the relevant future periods.
Common-size or other comprehensive ratio statements if appropriate.

Be sure you format your spreadsheets correctly, including dollar signs where relevant, commas between the thousands place and the hundreds place, leading zeroes in front of decimal fractions, and complete explanations.
FINANCE 700, Summer, 2008--Week 1 May 27, 29--BASICS OF VALUATION:

FINANCE PROCEEDS USING FINANCIAL CASH FLOWS
VALUATION IS THE CRUX OF FINANCE
SUCCESSFUL ENTREPRENEURSHIP CREATES VALUE;
UNSUCCESSFUL ENTREPRENEURSHIP DESTROYS VALUE.

Discounting future cash flows to present value: spot rates, forward rates, the Term Structure of Interest Rates, Inflation: nominal and real rates. BD Chapters 1, 4, 5

Portfolio Theory and Capital Asset Pricing Model. BD Chapters 2, 3.

The market interactions of the personal time-preference rates of market participants lead to the emergence of a system of market interest rates. The mathematics of compounding and discounting are fundamental to all financial decisions. Computations of present values of streams of future cash flows are stressed.

Freedom in economic transactions is essential to the allocative efficiency of markets. It is the existence of the system of market interest rates within the structure of the private-property economy which allows the making of rational financial decisions within the firm and the allocation of goods within a society across time in accordance with the values of the people who compose the society. The relationship between risk and required return is explored, and the term structure of interest rates is investigated.

A bond Yield to Maturity is neither a Spot Rate nor a Forward Rate, but is an Internal Average Rate of Return. A **Spot Rate** $0s_n$ relates the present moment to one moment only at some time in the future which is $n$ periods from now:

$$PV_0 = \frac{CF_n}{(1 + 0s_n)^n}$$

A **Forward Rate** $nf_n$ relates one moment at the beginning of the $n^{th}$ period with one moment at the end of the period; if the forward rates cover one year each, a stream of $n$ interest factors is required to discount a future cash flow $n$ years in the future to today:

$$PV_0 = \frac{CF_n}{(1 + 0f_1)(1 + 1f_2)(1 + 2f_3)(1 + 3f_4) \ldots (1 + n-1f_n)}.$$  

An Average Annual Discount Rate, $k = 0k_n$ relates the present moment 0 to one moment $n$ periods from now and is applied annually, so that it is raised to the $n^{th}$ power. This is the ordinary discounting that you have seen thus far.

$$PV_0 = \frac{CF_n}{(1 + k)^n}. \text{ Note that } k \text{ is NOT equal to } 0s_n.$$  

A risk-adjusted rate $ke = \text{risk-free rate } R_f + \text{risk adjustment } R_{risk}$.

**CAPM:**

$$ke = R_f + \beta (E[R_m] - R_f) + \text{NSRA}$$

$$ke = kd + k_{risk} \text{ also, where } kd = R_f + k_{bond\ risk}.$$  

$$k^{*} = \theta kd (1 - \tau) + (1 - \theta) ke$$

$ke$ is used to discount dividends and LFCFE's; $k^{*}$ is used to discount UFCFF's.
Goal of the Firm:

to increase as much as possible the present wealth of the shareholders, PWE_0.

Maximize PWE_0 = NCDE_0 + VE_0.

The present wealth of the shareholders takes full account of the risk the shareholders perceive and bear in owning the equity of the firm.

This is accomplished by choosing the alternative course of action with the largest Net Present Value, NPV_0 for the entire lifetime of the firm, after subtracting the cash outlay necessary to purchase the course of action IVS_0 < 0 and after discounting the forecasted future incremental net after-tax cash flows at the risk-adjusted cost of capital; T = terminus value, the last explicitly-forecasted time point and the location of the Terminal Value. The Terminal Value TV_T is the value at time T of all of the remaining cash flows beyond time T in the future for the remaining lifetime of the firm: Using the Gordon constant-perpetual-growth model, TV_T = CF_{T+1} / (k – g_\infty). For individual assets, we generally stop computing cash flows for the individual asset at the asset’s lifetime T.

The amount of the Net Present Value of a newly-embarked-on course of action is immediately added to the Value of Equity: ΔVE_0 = NPV_0.

So choosing the course of action with the largest NPV automatically increases the Present Wealth of the Equity Suppliers as much as is possible. This increment to the value of equity is made immediately upon the decision to pursue that course of action, before any physical action has been taken.

The after-tax marginal (additional with the project) differential (“with – without”) net cash flows, including opportunity costs, ignoring sunk costs, which we discount are the following:

1. **Gordon constant-perpetual-growth model.**

   Not only does Gordon over-estimate equity value because of its assumption of constant perpetual growth, because of its assumption of continuous improvement in the profitability of available investment projects year after year, but also because it assumes that larger firms will have more such superior investment opportunities because they are larger, so it is profitable to become large immediately by purchasing investment projects with negative Net Present Values which will add to firm size, even though each reduces its value: but the larger firm size creates more highly-profitable investment opportunities which would not be available if the firm were not so large, and these new high-profit opportunities overcome the loss due to the previously-accepted negative-NPV projects. The Gordon model values size for its own sake, and it recommends investment into projects with IRR’s down to the earnings-price ratio (E/V), which is smaller than the cost of capital k_e. These projects with IRR < k_e have NPV < 0, so these projects each lose value; but they make the firm larger, and the larger firm will have access next year to very-high-yielding new investment projects, which it would not have had available if it had not purchased these low-yielding projects this year. It is possible that the Gordon model causes up to a 30% over-valuation of the equity of the firm. You should multiply the value of the Gordon model by a factor such as 0.90 or even smaller to adjust for this over-valuation. This is especially true if you are using the Gordon model to estimate the Terminal Value_T in a business valuation exercise.

To value stock: \[ P_t = \frac{d_{t+1}}{(k_e - g_\infty)} \]

To value equity: \[ VE_0 = N_{shs} d_1 / (k_e - g_\infty) \]; \[ TV_T = CF_{T+1} / (k - g_\infty) \]

To estimate the cost of equity capital: \[ k_e = \left[ \frac{d_1}{P_0} \right] + g_\infty \]
THE GORDON MODEL OF CONSTANT, PERPETUAL GROWTH AT THE RATE $g_\infty$
BEGINNING AT $t_0$ FOR ALL-EQUITY, UNLEVERED FIRMS:

\[
P_0 = \frac{d_1}{(k_e - g_\infty)} = \frac{d_0(1+g_\infty)}{(k_e - g_\infty)}
\]

\[
k_e = \frac{d_1}{P_0} + g_\infty
\]

The dividends $d_t$ begin at $t_0$ to grow at the constant rate $g_\infty$ and they grow at that constant rate forever; hence the model is often called the "constant perpetual growth model." The dividend $d_1$ at $t_1$ is the first dividend determined by the constant perpetual growth rate $g_\infty$.

If the dividends begin to grow at their constant perpetual growth rate at a different moment than $t_0$, then the model is different.

- $P_0 =$ the present value of the future dividends = the price now
- $d_0 =$ the current dividend payable this year
- $d_1 =$ the dividend payable after one year from today
- $g_\infty =$ the constant perpetual growth rate of dividends and of all income items
- $b =$ retention rate, the portion of income retained and reinvested = NIAT - Dividends;
- $r =$ the average rate on new investment earned by the invested capital = ΔNIAT/IVSt.

$P_0 =$ the price now = $V_0 = \frac{d_1}{(k_e - g_\infty)}$

$V_T =$ general expression for present value of stock = $d_{T+1} / (k_e - g_\infty)$

$k_e =$ cost of equity capital = $d_1 / P_0 + g_\infty$

$g_\infty = br$ for all-equity firm

THE GORDON MODEL FOR LEVERED FIRMS USING DEBT-CAPITAL FINANCING

$M =$ the ratio of the market value of retained earnings to the dollar size or book value of retained earnings. If all investments have positive Net Present Values, then $M > 1.0$.

If we relax the assumption that the firm is all-equity financed by allowing the firm to adopt a constant debt/equity ratio, $(VD/VE)$, it can be shown that, if $M = 1.0$:

\[
g = r b + b [r - kd (1-t_c)] (VD/VE).
\]

Using debt financing can (but will not always) increase the growth rate, when the rate of return on the assets purchased exceeds the interest rate paid on the debt. This situation is called “favorable financial leverage”. When financial leverage is favorable, use of debt will raise the return on equity above the return on assets. Financial leverage always increases the risk borne by the equity holders.

If some financial leverage is employed, then $VD > 0$, and $(VD/VE) > 0$, and if $M > 1.0$, then

\[
g = \frac{\Delta E}{E} = r b + [r - kd (1-t_c)] M b (VD/VE).
\]

The above is the general case.

This general case dissolves to the all-equity Gordon model if $VD = 0$ because then the second term vanishes and

\[
g = r b = b r.
\]

If the least-profitable accepted project has IRR = kf*, then the average rate of return on new investment, $r > kf*$, and $M > 1.0$.

If $M = 1.0$, which is what occurs when the average rate of return on new investment $r$ equals the cost of capital, meaning that some projects are accepted whose rate of return is less than $kf*$, then

\[
g = r
\]

$\quad b + b [r - kd (1-t_c)] (VD/VE)$, only when $M=1.0$
GORDON GROWTH MODEL:
CONSTANT PERPETUAL GROWTH OF CASH FLOWS AT \( g_\infty \) BEGINS AT \( T \)

\[ CF_{T+1} = CF_T (1 + g_\infty) \]

is the first cash flow determined by the constant-growth process.

\( V_T = \text{Present Value at Time } T \) of all subsequent cash flows beginning one period hence (at \( T+1 \)) and growing at \( g_\infty \) for the life of the firm, discounted at \( k \) back to \( T \) from wherever they occur in the future out to infinity:

\[
V_T = \sum_{t=1}^{\infty} \left[ \frac{CF_t}{(1+k)^t} \right] = \frac{CF_{T+1}}{(k - g_\infty)}
\]

All assets are valued by discounting back to the present (time “0”) the infinite future stream of cash benefits at each future year “\( t \)’’ the asset will provide to its owner from next year (the “ex-dividend assumption” of Finance) to the end of its existence. If the cash flows we discount are dividends per share \( d_t \), then the value we compute is the price now of one share of common stock, \( P_0 \). If the cash flows we discount are the total dividends paid to all shares \( \text{DIV}_t \), where \( N = \text{number of shares outstanding} \), then the value we compute is the value now of equity of the firm, \( VE_\infty \). If the cash flows we discount are the LFCFE\( _t \)’s, then the value we compute is also the value now of equity of the firm \( VE_\infty \).

It is inconvenient to evaluate an infinite series of future discounted cash flows. So we search for an approximate method which is more convenient; we transform the infinite series of terms into a finite “closed form” if we can. One type of infinite series which we can correctly convert into a closed form is a series in which each subsequent term is smaller than the preceding term. If the subsequent term is enough smaller than the preceding term, then the infinite series, even though it is infinite, will converge to a finite sum and will have a closed form. The series consists of discounted future cash flows, in which each cash flow is discounted at the discount rate \( k \) from the time point where it occurs back to the present, by dividing the cash flow by the discount factor \( (1 + k)^t \), where \( t \) is the number of years in the future when the cash flow occurs. If each year, the cash flow in the numerator of the term grows \( g_\infty \) larger than the previous year’s cash flow, and if \( k > g_\infty \), then the series will converge to a finite sum and we can find the closed form which is equivalent to the infinite series.

The closed form solution, the Gordon Model, is:

\[ V_0 = \frac{CF_1}{(k - g_\infty)} \]

or in general, \( V_T = \frac{CF_{T+1}}{(k - g_\infty)} \)

<table>
<thead>
<tr>
<th>Time Points:</th>
<th>-1</th>
<th>0</th>
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<td>Cash Flows:</td>
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<td>\ldots</td>
</tr>
</tbody>
</table>

Values: \( V_T = \frac{CF_{T+1}}{(k-g_\infty)} \)

Values: \( V_0 = \frac{CF_1}{(k-g_\infty)} = \frac{CF_0 (1+g_\infty)}{(k-g_\infty)} \)

Values: \( V_2 = \frac{CF_2}{(k-g_\infty)} \)

Notice that \( CF_T \) is not determined by the constant growth rate \( g_\infty \); rather, \( CF_T \) is determined by some other process and generally, the growth rate between \( CF_{T-1} \) and \( CF_T \) is larger than \( g_\infty \). Notice that \( CF_{T+1} \) is the first cash flow determined by the constant growth rate \( g_\infty \) which began at \( T \). Notice that all subsequent cash flows following \( CF_T \) are determined by the constant growth rate \( g_\infty \). Notice the one-period gap between the time of valuation \( T \) and the \( T+1 \) identity of the cash flow in the numerator of the Gordon Equation.

The Gordon Growth Model is a very-widely-used model in estimating the value of common stock and in estimating the value of a business firm at time \( T \) when the cash flows grow forever beginning at time \( T \) at a constant rate \( g_\infty \). But the cash flow at time \( T \), \( CF_T \), the last cash flow not determined by the constant-growth process, is determined by some other, larger growth rate than \( g_\infty \), or by some explicit forecasting process. Time \( T \) is the “terminus” of the period of explicit forecast, the last time point for which an income statement or dividend is explicitly forecast. In the context of valuing a business firm, the Gordon Model is the most-commonly used model for the Terminal Value at time \( T \), the value the firm will
have at time \( T \), the end of the period of explicit forecast. The Gordon Model estimates the value of the cash flow stream at time \( T \), the beginning of the infinitely-long period of constant growth, and it uses the amount of the cash flow one period later at \( T+1 \), the first cash flow determined by the constant-growth process, which is \( CF_{T+1} = CF_T (1 + g_c) \).

\[
V_T = \frac{CF_{T+1}}{(k - g_c)}
\]

Because the Gordon Model assumes that the lifetime of the firm is infinite, and that the constant growth goes on forever, the Gordon model tends to overestimate the true value of the firm at any time point. The Gordon model is the closed-form solution to the convergence of the infinite series of discounted future cash flows. “Convergence” means that the infinite series of terms has a finite sum. During the entire infinitely-long process, the growth rate \( g_c \) does not change, so each cash flow at time \( t \) is the product of the first cash flow \( CF_T \) which is itself not determined by the constant growth process, multiplied by the factor \( (1+ g_c)^n \), where \( n \) is the number of periods between the cash flow’s time point and the initial time point \( T \); that is, \( n = t - T \).

The model is the same, whether we are valuing a share of stock paying a constantly-growing dividend per share, or valuing a firm generating a constantly-growing overall cash flow. We merely use the discount rate corresponding to the identity of the cash flow being valued, and we use the cash flow one period futureward from the moment we value.

1. For example, suppose we are valuing a share of stock, and we believe that five years from now the dividends will begin to grow at a constant perpetual rate \( g_c \), but between now and then they will grow at variable rates which are larger than \( g_c \); then:

\[
P_0 = \frac{d_1}{(1+ke)^1} + \frac{d_2}{(1+ke)^2} + \frac{d_3}{(1+ke)^3} + \frac{d_4}{(1+ke)^4} + \frac{d_5}{(1+ke)^5} + P_5/(1+ke)^5
\]

where \( P_5 \) is the price of the stock at time 5, and that depends on all of the subsequent dividends beyond time 5 beginning at time 6 and going on to infinity.

We note that the Gordon Model gives

\[
P_5 = d_6/(ke - g_c) = d_5 (1+g_c) / (ke - g_c),
\]

so we get

\[
P_0 = \frac{d_1}{(1+ke)^1} + \frac{d_2}{(1+ke)^2} + \frac{d_3}{(1+ke)^3} + \frac{d_4}{(1+ke)^4} + \frac{d_5}{(1+ke)^5} + \left[ \frac{d_5 (1+g_c) / (ke - g_c) }{(1+ke)^5} \right]
\]

2. For example, suppose we are valuing an entire business firm now using the value of its equity \( VE_0 \), and we believe that five years from now, the LFCFE's will begin growing at a constant rate \( g_c \) forever, but that prior to then, the growth rates will be larger than \( g_c \) and variable from year to year; then:

\[
VE_0 = \frac{LFCFE_1}{(1+ke)^1} + \frac{LFCFE_2}{(1+ke)^2} + \frac{LFCFE_3}{(1+ke)^3} + \frac{LFCFE_4}{(1+ke)^4} + \frac{LFCFE_5}{(1+ke)^5} + \frac{VE_5}{(1+ke)^5}
\]
We note that the Gordon Model gives

\[ VE_5 = \text{LFCFE}_0/(ke - g_x) = \text{LFCFE}_5 (1+g_x) / (ke - g_x) \]

so we get

\[
VE_0 = \frac{\text{LFCFE}_1}{(1+ke)^1} + \frac{\text{LFCFE}_2}{(1+ke)^2} + \frac{\text{LFCFE}_3}{(1+ke)^3} + \frac{\text{LFCFE}_4}{(1+ke)^4} + \frac{\text{LFCFE}_5}{(1+ke)^5} + \left[ \frac{\text{LFCFE}_5 (1+g_x) / (ke - g_x)}{1+ke} \right] / (1+ke)^5
\]

Note that the constant perpetual growth rate \( g_x \), which appears in the denominator of the Gordon Model as a subtraction from the cost of capital \( k \), and which begins only at time \( T \) where the Gordon Model is applied, is DIFFERENT FROM the prior growth rate of cash flows between \( T-1 \) and \( T \), and also different from the other growth rates prior to time \( T \). In real life, the constant perpetual growth rate \( g_x \) cannot be larger than 8.0% in my opinion, because if it were, the company would in thirty-five years be larger than any other company in the world, and most national economies as well because of “the magic of compound growth.” If you use in the Gordon Model a higher growth rate than the actual constant perpetual growth rate which begins only when the firm reaches maturity, you will calculate an unrealistic and impossibly-large Terminal Value and your calculations will be extremely wrong.

INNER WORKINGS OF THE GORDON MODEL: \( g_x = \text{br} \)

We use small letters, such as “\( d \)” or “\( e \)” to represent –per share amounts, and we use capital letters, such as “\( D \)” or “\( E \)” to represent amounts for the entire firm, with \( D = Nd \), and \( E = Ne \), where \( N \) is the number of common shares outstanding. \( E = \text{NIAT} \), and \( D = \text{DIV} = \text{LFCFE} \). We term the amount of earnings not paid out as dividends the “cash retention of the firm”, or “\( \text{crf} \)” on a –per share basis, and “\( \text{CRF} \)” on a total firm basis. \( \text{crf}_t = e_t - d_t \); \( \text{CRF}_t = \text{NIAT}_t - \text{DIV}_t \). The firm must either pay out or retain and reinvest all earnings: \( e_t = d_t + \text{crf}_t \); and \( \text{NIAT}_t = \text{CRF}_t + \text{DIV}_t \).

Whether we are valuing a single share of stock or the entire equity of the firm, the cash flow discounted in the Gordon Model is a dividend \( d_t \), or the amount of dividends paid by the firm \( \text{DIV}_t = Nd_t \) (where \( N \) is the number of shares outstanding), or the amount of dividends the firm could pay if it wished, \( \text{LFCFE}_t \). In the Gordon Model, \( \text{LFCFE}_t = \text{DIV}_t = Nd_t = N(e-crf) \).

Because the Gordon Model deals with the time when the firm is mature; i.e., following its rapid growth phase or its variable-rate growth phase and has now settled down to a stable growth rate, the market into which the firm sells during its maturity is also stable, and the market share the firm holds is constant. There are no more technological innovations to production during the stable lifetime, and the firm has learned all there is to know about the operation of its capital assets, so no further progress is made in generating sales with assets (\( S/TA \)) or in generating income from sales (\( \text{NI/S} \)), and we see that the operating efficiency of the firm is constant during its mature lifetime. Because the capital structure of the firm is also constant during its maturity, we can neglect further changes in the capital structure and consider the firm an “all equity” firm. The Gordon Model strictly can deal only with all-equity firms.

With all this stability during the phase described by the Gordon Model, we have a constant structure of the income statement. First, we assume that the dividends paid are a constant fraction of the income earned by the firm, or the “earnings.” This fraction is called \( b \), the “plowback rate,” the fraction of \( \text{NIAT} \) paid out in dividends, and it is the same whether we consider all dividends \( \text{DIV} = Nd \) and all income \( \text{NIAT} = Ne \), or the dividend per share \( d \) and the earnings per share \( e \), where \( N \) is the number of common shares outstanding.

\[ p = \frac{\text{DIV}}{\text{NIAT}} = \frac{d}{e}, \text{ a constant across time} \]

Because we are in stable operating mode and we are neglecting debt, all income not paid out as dividends is retained and reinvested by the firm. \( \text{NIAT}_t - \text{DIV}_t = \text{CRF}_t \). The portion of income retained and reinvested is called \( b \) in the Gordon Model, the “retention rate” or the “plowback rate,” the same as the factor “\( R \)” in the Sustainable Growth Model.

\[ b = \frac{\text{CRF}}{\text{NIAT}} = \frac{e}{e} = (1-p) = \frac{(e-d)}{e} = \frac{\text{NIAT} - \text{DIV}}{\text{NIAT}}, \text{ a constant across time} \]

So \( p + b = 1.0 \).

Since \( \text{TCC} = \text{IVS} \), \( \text{IVS} = \text{CRF} \) and \( \text{ivs} = \text{crf} \), on either the total-firm or the –per share basis.

Remember, the basic assumption of the Gordon Model is that dividends, either –per share or total, grow forever at the rate \( g_x \). But now we see that dividends are a constant fraction \( p \) of earnings, so we see that earnings must also grow.
forever at the constant rate g. And the other constant portion of earnings, retained cash that is reinvested crft = et - dt, or CRF = NIAT - DIV, must also grow at the constant rate g. And since investments is the cash retained by the firm, ivst = crft, and IVS = CRF, we see that the amount invested by the firm ivst grows also at the constant rate g.

Now we ask: How does the firm cause its income and dividends to grow at the rate g? We are not issuing any new debt, and we are not selling any new equity, so the total cash available for investment is crft the cash retained from operations, and ivst = crft = e - d. The growth in earnings, and the consequent growth in dividends, must come about from the return earned by the investments. Let us call this rate of return earned on the investments r.

\[ r = \text{the average rate of return earned on the investments each year} \]

So, the additional income earned in each year t compared with the income earned in the prior year t-1, is the return provided in year t by the investment outlays in the prior year, and the investment is a portion of income ivst = be:

\[ t-1 \Delta e_t = e_t - e_{t-1} = r \left( ivs_{t-1} \right) = r be_{t-1} \]

and \[ e_t = e_{t-1} \left( 1 + r \right) = e_{t-1} + t-1 \Delta e_t = e_{t-1} + r be_{t-1} \]

and we see that \[ r = t-1 \Delta e_t / ivs_{t-1} \]

But \[ t-1 \Delta e_t = e_t - e_{t-1} \left( 1 + g_\infty \right) \]

And \[ e_t = e_{t-1} + rb e_{t-1} = e_{t-1} (1 + g_\infty) = e_{t-1} + g e_{t-1} \]

So \[ g = br \]

The growth of earnings and dividends comes about because of the portion of income retained and reinvested to earn the rate of return r. The larger the portion of income retained and reinvested, the larger is the rate of growth of dividends. The higher the rate of return earned by the investments, the faster the growth of the company.

Because \[ r = \text{the average rate of return earned by the entire set of investments, we see that some investments earn} \]

much higher rates of return than r, and some investments earn smaller rates of return than r. But remember the first rule of finance: seek to increase the wealth of the shareholders by investing to earn a rate of return no smaller than the rate of return required by the investors. This means that the least profitable investment project we invest in has a rate of return equal to ke, the cost of capital required by the investors, the rate of return the investors require because of the risk they perceive in owning our stock. (If the firm has debt, then the minimum acceptable rate of return is kF* = Θkd* + (1 - Θ) ke, the weighted average cost of capital.) So \[ r > ke \]

It is the profitable investment of crft = ivst into projects all having IRR > ke (and remember if the IRR > k, then NPV > 0 ) which causes the firm to grow. So the Gordon model is completely consistent (as far as we can see now) with correct Financial theory: profitable investments create more income next year than we had last year, and this larger income allows the payment of larger dividends year after year because each year we invest into a new set of profitable projects.

**GROWTH RATES OF LEVERED FIRMS**

**Definitions of terms.** The time-subscript t indicates the moment in time at which a value is computed. The subscript 0 indicates the value at the present moment. Values are computed one period before the first cash flow under consideration.

VDt = the market value of debt at time t, taking account of the current market interest rate,

VEt = the market value of equity at time t = Number of shares X Price per share at time t,

VFt = VDt + VEt, = the value of the firm at time t,

VDt / VFt = the capital structure ratio, or debt ratio, of the firm at time t = the portion of capital raised from debt = 1 - (VEt / VFt); to be held constant for now,

(VEt / VFt) = the equity ratio, the portion of capital raised from equity = 1 - (VDt / VFt),

(VDt / VEt) = the ratio of debt to equity of the firm, likewise to be held constant for now,

tc = the marginal corporate income tax rate,
kd = the rate of return required by debt suppliers in the period beginning at time t,
k\* = kd(1 - t\) = the cost of debt capital for the period beginning at time t,
ke = the rate of return required by equity suppliers for the period beginning at time t= the cost of equity capital beginning at time t,
kf\* = the firm’s weighted average cost of capital, used to compute the Net Present Value of each investment project = (VD/VF) kd\* + (VE/VF) ke,
CDE\t = the cash distributed to equity holders = total dividend paid by the firm at time t,
N = the number of shares outstanding,
E\t = the total earnings of the firm after taxes during the period ending at time t,
\AE = change in earnings from one time period to the next,
E\t+1 = E\t + \AE,
e\t = the earnings per share of the firm after taxes during the period ending at time t = E\t / N,
d\t = the dividend per share being paid at time t = CDE\t / N,
IVS\t = the investment expenditure of the firm at time t,
ii = the per-share portion of investment expenditures at time t = IVS\t / N,
P\0 = the current price of the stock, per share, at time 0,
g = the growth rate of dividends across the entire future time; g = br, if VD = 0;
g = also the growth rate of earnings, investment expenditures, and market price of the stock from period to period; everything grows at the rate g because the retention rate b is constant,
b = the earnings retention rate of the firm, the portion of earnings retained and reinvested = ( IVS\t / E\t ),
b E\t = the amount of earnings retained and reinvested at time t; b E\t = IVS\t,
p = (1 - b) = the payout fraction of income, that portion of income paid out as a dividend at time t; p = (1 - b) = (CDE\t / E\t ),
r = the average rate of return after tax on the set of new accepted investment projects; because the least-profitable accepted project has an internal rate of return no smaller than kf\*, the value of r > kf\*.
r is computed as the weighted-average of the internal rates of return of all the accepted projects, where each project’s internal rate of return is weighted by the fraction of its time-zero outlay to the total of all time-zero outlays, which is the ratio of the investment in the project divided by the total investment outlay in all the projects, assuming investment only at t\0.
M = the ratio of the market value of retained earnings to the dollar size or book value of retained earnings. If all investments have positive Net Present Values, then M > 1.0.

THE GORDON-SHAPIRO MODEL OF CONSTANT-GROWTH OF AN ALL-EQUITY FIRM

The current market value of a share of common stock is equal to the present value of all future dividends, discounted at the cost of equity capital.

The cost of equity capital ke is that rate of return the stock market expects to receive in order to compensate it for the use of funds and the risk associated with the future dividend stream. The cost of equity capital is the rate of discount that equates the present value of all future expected dividends per share to the present price of the common stock. The cost of equity capital is determined by the term structure of interest rates of risk-free loans (the time preferences of the market participants) and the risk borne by the equity holder of the stock due to the uncertainty of the future dividend stream. The variability of the future earnings of the firm, which affects the variability of the dividend stream, depends on the business risk of the firm and the degree of financial leverage employed. The variability of future earnings also depends on the rates of inflation during different periods of time in the future.

Business risk arises from fluctuations in sales magnified by the degree of operating leverage of the firm. Business risk is measured by fluctuations in the operating income of the firm, earnings before interest and taxes. Business risk is larger than the fluctuations in sales revenue.
Fluctuations in the firm’s operating income are magnified by financial leverage, which is caused by interest-bearing debt. Financial leverage causes the fluctuations in the returns to equity suppliers to be larger than the fluctuations of operating income. Fluctuations in the returns to equity suppliers cause equity risk. Because of financial leverage, equity risk is always larger than business risk if the firm has any debt. The cost of equity capital $ke$ of a leveraged firm (which is caused by the equity risk) is larger than the firm’s overall required rate $kf$ (which is caused by the business risk) whenever there is debt.

The anticipated growth rate of dividends for any firm depends on the opportunities for investment perceived by the managers and the financing strategies of the firm. The growth rate of dividends depends on the firm’s rate of return on investment and on its rate of reinvestment out of operating cash flows.

The Gordon-Shapiro constant-perpetual growth rate model applies only to unlevered (“all-equity”) firms, which have no debt and so are financed entirely by equity. The model computes the value of equity $VE_0$, or the price per share $P_0$, based on the next period’s total dividend $CDE_1$ or dividend per share $d_1$, the cost of equity capital $ke$, and the growth rate $g$.

$$VE_0 = \frac{CDE_1}{(ke - g)}$$

$$P_0 = \frac{d_1}{(ke - g)}$$

$$P_0 \times \text{Number of shares} = VE_0$$

or alternatively,

$$P_0 = \frac{[(1-b) e_1]}{[ke - b \ r]}.$$ 

The change in earnings $E_2 - E_1 = \Delta E = g E_1$, that is, the increment to earnings is the growth rate multiplied by the previous earnings. Also, $E_2 = E_1 + \Delta E = E_1 (1 + g)$.

In the Gordon model, the growth rate $g$ is the product of the retention rate $b$ and the average rate of return earned after taxes on new investment $r$. The growth rate is therefore given by:

$$g = b \ r$$

because, if the firm is all-equity financed and sells no new equity shares, the only source of investment capital is the retained portion of cash provided by operations. The amount of earnings retained is $(b \ E)$, and the average after-tax rate of return earned by the investment is $r$. Therefore,

$$E = r \ b \ E = g \ E$$

and

$$E_2 = E_1 (1 + r \ b)$$

The growth rate of net income for a period $g$ is equal to the product of the after-tax rate of return on new investment $r$ times the rate of earnings retention $b$. With the constant retention rate $b$, the growth rate of dividends is equal to the growth rate of net income. If the rate of return on new investment $r$ is constant from period to period, then the growth rate $g$ is constant from period to period.
ESTIMATING $k_e$, THE COST OF EQUITY CAPITAL

The Gordon model also provides a reasonable estimate of the cost of equity capital $k_e$:

$$ k_e = \left( \frac{d_1}{P_0} \right) + g $$

in which we see that the cost of equity capital is larger than the dividend/price ratio by the expected growth rate $g$ of dividends. Notice that there is a one-period gap between the dividend per share and the price: presumably $d_1 > d_0$ because of the growth of dividends at the rate $g$, so this ratio is larger than it would be if the dividend contemporaneous with the price were used. Notice that the stockholders appropriate all of the value arising from the growth to themselves by incorporating the entire growth rate into the cost of equity capital. The growth therefore benefits only stockholders and cannot benefit bondholders, except by reducing their risk.

Often, people have considered that a good estimator of the cost of equity capital $k_e$ is the earnings/price ratio ($\frac{e_0}{P_0}$). You should first be uneasy about this because the earnings contemporaneous with the price are used. As you know, discounting a future cash flow to present value usually includes a gap of one period between the present value and the future cash flow. We should first consider modifying the earnings/price ratio to be ($\frac{e_1}{P_0}$). Keep in mind this necessity of using the leading (i.e., future period) earnings in computing an earnings/price ratio. We know that the current price depends on future earnings, not at all upon current earnings, which already cannot be altered.

Sometimes, the Earnings/price ratio is a valid estimator of the cost of equity capital; but more often, it is not a good estimator at all because it ignores the investment outlays needed to bring about growth and the growth itself, which has, as we have seen above, an influence on $k_e$.

$$ k_e = \left( \frac{e_1}{P_0} \right) $$ is a good estimator of the cost of equity capital if the firm does not retain any cash from operations, so that $b = 0$, and therefore the firm pays all its earnings out as dividends and therefore does not grow, so that $g = 0$. We know already, of course, that if the firm does not grow, then $d_t = d_{t-1}$, and $d_t = e_t$. The earnings/price ratio is also a good estimator of the cost of equity capital if the firm engages in expansion, not growth. Recall that expansion is the situation in which the firm retains some earnings ($b > 0$) and reinvests them, but reinvests in too many projects, bringing the average rate of return on new investment $r$ down to the value of the weighted-average cost of capital $k^*$. So ($\frac{e_1}{P_0}$) is a good estimator of $k_e$ if $r = k^*$. So long as $r > k^*$, the earnings/price ratio under-estimates the cost of equity capital. Recall that expansion produces the same value of the firm as does no growth whatever. So only in the no-growth case is the earnings/price ratio a good estimator of the cost of equity capital.
LEVERED FIRMS USING DEBT-CAPITAL FINANCING

If we relax the assumption that the firm is all-equity financed by allowing the firm to adopt a constant debt/equity ratio, \( \frac{VD}{VE} \), it can be shown that:

\[ g = r b + b[r - kd (1 - t_c)] \left( \frac{VD}{VE} \right). \]

Using debt financing can (but will not always) increase the growth rate, when the rate of return on the assets purchased exceeds the interest rate paid on the debt. This situation is called “favorable financial leverage”. When financial leverage is favorable, use of debt will raise the return on equity above the return on assets. Financial leverage always increases the risk borne by the equity holders.

If some financial leverage is employed, then \( VD > 0 \), and \( \frac{VD}{VE} > 0 \), and for that degree of leverage, if \( r > kd (1 - t_c) \), then the second term of the above equation is positive, implying that the use of this degree of leverage increases the growth rate of earnings and dividends over that which would have occurred without the use of debt. This follows since the average rate of return on investment \( r \) has been assumed to be constant, independent of the amount invested. Assuming that \( r > kd (1 - t_c) \) is equivalent to assuming the marginal (which equals the average if the average is unchanging) return on investment is greater than the after-tax cost of debt capital, so that debt utilization creates residual equity earnings. Use of such favorable financial leverage increases the growth rate of earnings and dividends. This is because debt is inherently cheaper than equity due to its smaller risk and the deductibility of interest from taxable income. Use of debt allows purchase of more assets, and leaves greater residual earnings available after paying for the capital.

It is beneficial to the firm to borrow, for example, $1,000,000 at an 8.0% rate and then invest this million into a machine that earns an internal rate of return of 15%. The machine earns $150,000 income each year, and $80,000 of that is used to pay the interest on the debt, leaving $70,000 available for the increase in equity value that accompanies cash provided by operations. From this $70,000, the firm must pay the appropriate portion for dividends also.

If the firm employs debt and desires to maintain a constant debt/equity ratio of \( \frac{VD}{VF} \), where both debt and equity are measured at market value, a dollar of retained earnings may increase the market value of the common stock more or less than one dollar, depending on the yield of the investments being undertaken by the firm. If the investments earn a yield in excess of the weighted average cost of capital, the suppliers of equity capital will receive more than their required return (by a combination of dividends or capital gain), which is the cost of equity capital. They will therefore place a market value on the retained earnings in excess of the incremental book value: \( M \) dollars of market value for every dollar of retained earnings. If the investments earn a yield smaller than the weighted average cost of capital, the suppliers of equity capital will suffer a reduction in the combination of dividends or stock price, and their net receipt will be smaller than their required rate of return, and the value of \( M \) will be < 1.0, so that the increase in the market value of retained earnings will be smaller than the increase in book value.

We consider the situation in which the firm holds constant its debt ratio \( \frac{VD}{VF} \) and its debt/equity ratio \( \frac{VD}{VE} \). This is the usual situation. Changes in capital structure are complex and should not be combined with other decisions, but considered separately. We assume that the firm has already found its optimum capital structure (the value of \( \frac{VD}{VF} \) which minimizes the weighted-average cost of capital and maximizes the value of the firm). When the capital structure is held constant, and when we assume also that the firm does not sell new equity capital (which is also the more common situation), then the only source of additions to equity capital is cash retained from operations, or
“retained earnings”. So we are going to consider the cash added to the book value of equity in the form of retained earnings.

Remember that the entire Net Present Value of the investment in new plant and equipment is added to the value of equity. The Net Present Value is computed net of the need to pay interest (net of income tax effects) to the bondholders and the required rate of return to the equity holders, so the Net Present Value is a net gain to the equity holders above and beyond their required rate of return.

We are going to hold the capital structure constant. But we know that there will be an increase in the value of equity due to the Net Present Value of the new equipment purchased. To hold constant the capital structure means that new debt must be borrowed to increase the value of debt so that the ratio of the value of debt to the value of equity remains constant, and the ratio of the value of debt to the value of the firm (the capital structure ratio or the debt ratio) also remains constant.

Always keep in mind that, when capital structure is held constant, even when no new equity capital is issued, it is necessary to increase the total debt outstanding (that is, borrow more) to compensate for the increase in the value of equity occurring from successful operations which produce a new increment to the value of equity from newly-retained cash from operations (“new retained earnings”). This new borrowing is consistent with the maintained value of the capital structure ratio, so it should not be considered excessive. But if this new borrowing is not added to the value of debt, the capital structure ratio will decline to a more equity-heavy value than it should have.

Note also, please, that the accountant’s equity account “Retained Earnings” on the balance sheet is different: on the balance sheet the equity account Retained Earnings is the sum over the life of the firm (i.e., since its beginning) of Net Income After Taxes minus Dividends Paid, and subject to any adjustments made in the past.

We will have the same assumptions; i.e., constant capital structure, no new issuance of equity shares, etc. when we consider the Sustainable Growth Rate of the firm. Most of the time, of course, the firm does not issue new equity and does maintain its capital structure constant.

We need to consider the weighted-average cost of capital, “WACC”, or \( k_f^* \). Recall that

\[
  k_f^* = \frac{VD}{VF} \cdot kd \cdot (1 - t_c) + \frac{VE}{VF} \cdot ke,
\]

where
- \( VD \) = the market value of debt, taking account of the current market interest rate,
- \( VE \) = the market value of equity = Number of shares X Price per share,
- \( VF = VD + VE \),
- \( t_c \) = the marginal corporate income tax rate,
- \( kd \) = the rate of return required by debt suppliers,
- \( kd(1 - t_c) \) = the cost of debt capital,
- \( ke \) = the rate of return required by equity suppliers = the cost of equity capital,
- \( k_f^* \) = the firm’s weighted average cost of capital.

Suppose now that:
- \( IVS_0 = $1,000,000 \)
- \( IRR = r = 0.15 \)
- \( E_{t>0} = $150,000 \) before interest payments and dividends
- \( \frac{VD}{VF} = 0.40 \),
- \( \frac{VE}{VF} = 0.60 \),
kd = 0.08, 
ke = 0.12 
tc = 0.35, 

then \( kf^* = 0.40 \times 0.08 \times 0.65 + 0.60 \times 0.12 = 0.0928. \)

This means that the firm needs to provide $92,800 of the cash flow of $150,000 earned by the asset to service the capital suppliers. This leaves $57,200 to add to the value of equity as retained earnings.

If the investments in new assets earn a yield or rate of return larger than the weighted average cost of capital (which automatically means that the assets earn a rate of return larger than the cost of debt to the firm because the cost of equity is larger than the cost of debt), then the suppliers of equity capital, the stockholders, will receive more than their required return, which is the cost of equity capital. The equity suppliers will therefore place a market value on the retained earnings earned by the assets larger than the incremental book value of the retained earnings.

This means that the book value of the firm will, in the above example, increase by $57,200, but the market value of the firm will increase by a greater amount.

The ratio of the market value of retained earnings to the dollar size or book value of retained earnings is called \( M \). For example, in the situation above, the assets produce a book value of retained earnings of $57,200. However, because the projects have a positive NPV, \( M > 1.0. \) Suppose \( M = 1.20 \) due to the conditions in the capital markets. Then the increase in the Value of Equity = \( 1.20 \times 57,200 = 68,640. \)

We now derive the equation for the growth rate of dividends of a firm that has debt and is therefore “leveraged” and maintains a constant capital structure ratio \( \frac{VD}{VE} \) while it retains the fraction \( b \) of its income for reinvestment into assets having an average rate of return \( r. \) kd is the rate of return required by the debt holders, and \( t_c \) is the corporate income tax rate.

The total investment is the sum of debt issued plus retained earnings:

\[
IVS = \Delta VD + bE = (VD/VE) (M) (b) (E) + bE. 
\]

If each dollar of investment earns \( r, \) the additional earnings are:

\[
\Delta E = (r) (IVS) - \text{Interest on Debt} 
\]

so

\[
\Delta E = r [ bE + (VD/VE) (M) (b) (E)] - kd (1-t_c) (VD/VE) (M) (b) (E) 
\]

and

\[
\Delta E = r b E + [r - kd (1-t_c)] M b E (VD/VE). 
\]

The above is the incremental earnings \( \Delta E. \) This is related to the growth rate of earnings \( g. \)

\[
E (1+g) = E + \Delta E 
\]

and

\[
1+g = 1 + \Delta E/E 
\]

so that

\[
g = \Delta E/E = r b + [r - kd (1-t_c)] M b (VD/VE). 
\]

The above is the general case.
This general case dissolves to the Gordon model if \( VD = 0 \) because then the second term vanishes and

\[
g = r \cdot b = b \cdot r.
\]

If \( M = 1.0 \), which is what occurs when the average rate of return on new investment equals the cost of capital, meaning that some projects are accepted whose rate of return is less than \( kf^* \), then

\[
g = r \cdot b + b \left[ r - kd \left(1-t_c\right) \right] \left( VD/VE \right), \text{ only when } M=1.0
\]

2. **The Solomon Model is more conservative and does not overstate value.**

\[
VE_0 = \left[ E - I \right]/k + rI/k^2
\]

- \( E \) = current earnings
- \( I \) = current investment
- \( k \) = cost of capital
- \( r \) = average rate of return on new investment

3. **Sustainable Growth Model: reasonably accurate, but valid for only one period at a time.**

Capital assets are heterogeneous; they can interact in different ways to achieve a given end. Different quantities of different assets can interact to improve performance. A plethora of alternatives is available, some of which are better than others. (Ludwig Lachmann)

The Sustainable Growth Model depicts the speed, \( g^* \), with which sales can grow with specified operating efficiency and present assets. To improve operating efficiency or to grow faster than the present resources can attain requires purchasing new assets which require financing. To grow faster with constant operating efficiency than the \( g^* \) (which is based on the present asset structure) rate requires also purchasing new assets which require financing. The financing is described as an “absorption of cash.”

\[
g^* = PRAT^ = \text{growth rate of sales under normal circumstances}
\]

- \( P \) = profit margin on sales = \( NI/S = \text{Net Income after Tax} / \text{Sales Revenue} \)
- \( R \) = retention rate = \( CRF/NI = \text{Cash retained by the firm} / \text{NIAT} = (NI – \text{Dividends}) / NI \)
- \( A \) = total asset turnover = \( S/TA = \text{Sales} / \text{Total Assets} \)
- \( T^\wedge \) = assets to b-o-p equity leverage ratio = \( TA/NW_{bop} = \text{Total Assets} / \text{Beginning-of-period equity} \)

If actual \( g > g^* \), the firm must have either improved its operating efficiency or absorbed cash.
If actual \( g < g^* \), the firm must have reduced its operating efficiency or disgorged cash.
Goal of the Firm:

to increase as much as possible the present wealth of the shareholders, \( PWE_0 \).

Maximize \( PWE_0 = NCDE_0 + VE_0 \).

The present wealth of the shareholders takes full account of the risk the shareholders perceive and bear in owning the equity of the firm.

This is accomplished by choosing the alternative course of action with the largest Net Present Value, \( NPV_0 \) for the entire lifetime of the firm, after subtracting the cash outlay necessary to purchase the course of action \( IVS_0 < 0 \) and after discounting the forecasted future incremental net after-tax cash flows at the risk-adjusted cost of capital; \( T = \) terminus value, the last explicitly-forecasted time point and the location of the Terminal Value. The Terminal Value \( TV_T \) is the value at time \( T \) of all of the remaining cash flows beyond time \( T \) in the future for the remaining lifetime of the firm: Using the Gordon constant-perpetual-growth model, \( TV_T = \frac{CF_{T+1}}{k - g} \). For individual assets, we generally stop computing cash flows for the individual asset at the asset’s lifetime \( T \).

The amount of the Net Present Value of a newly-embarked-on course of action is immediately added to the Value of Equity: \( \Delta VE_0 = NPV_0 \).

So choosing the course of action with the largest NPV automatically increases the Present Wealth of the Equity Suppliers as much as is possible. This increment to the value of equity is made immediately upon the decision to pursue that course of action, before any physical action has been taken.

The after-tax marginal (additional with the project) differential (“with – without”) net cash flows, including opportunity costs, ignoring sunk costs, which we discount are the following:

1) \( \Delta FCF^*_t = \) differential/incremental net after-tax operating cash flow to the firm at time \( t \);

\[
\Delta FCF^*_t = (AS_t - AC_t - \Delta D_t) (1 - \tau) + \Delta D_t
\]

taking account of requirements of new net working capital \( \Delta NWC_t < 0 \) and releases of net working capital at the end of a machine’s lifetime \( \Delta NWC_t > 0 \); taking account of cash outlays to purchase the machinery initially \( IVS_0 < 0 \) and refurbish or clean it during its lifetime \( IVS_t < 0 \), and any other outlays required, such as cleaning the site \( IVS_T < 0 \) or selling the scrap \( SV_T > 0 \).

2) \( LFCFE_t = \) the leveraged free cash flow available to equity holders at time \( t \); the number of dollars the firm can afford to pay in dividends without adversely affecting the future growth of the firm.

\[
LFCFE_t = NIAT_t + DEPR_t + AMORT_t - IVS_t - \Delta NWC_t - PP_t + NDC_t + NPS_t - Pfd Divs_t
\]

\[
LFCFE_t = \text{Leveraged Free Cash Flow to Equity}_t =
NIAT_t + \text{Noncash charges}_t - IVS_t - \Delta NWC_t + \Delta NDC_t + NPS_t - Pfd Divs_t
\]

3) \( UFCFF_t = \) the unleveraged free cash flow available to all capital suppliers at time \( t \).

\[
\text{Unleveraged Free Cash Flow to the Firm}_t = UFCFF_t =
NIAT_t + DEPR_t + AMORT_t + Int_t (1 - \tau) - IVS_t - \Delta NWC_t
\]

UFCFF can be used to pay debt principal, debt interest, dividends, and purchase common stock in the market.
There are four types of decisions:

1) **Investment Decision**—selecting the best capital asset available to the firm:
   \[
   \text{NPV}_0(\text{Proposed Asset}) = \text{NPV}_0 \text{ of } \Delta C_f^* \text{ through } \Delta C_f^*_T \text{ discounted at } k_f^*,
   \]
   assuming \( \Theta \) constant; \( \Delta C_f^*_t \) = the after-tax incremental (or “differential”) net cash flow from operations (i.e., the cash flow of the firm with the new investment minus the cash flow of the firm without the investment), including any necessary outlays \( (IVS_t < 0) \), and including any necessary new differential net working capital \( (\Delta NWC_t < 0) \), and including any end-of-lifetime cash inflows such as salvage value \( (SV_T > 0) \), or outflows such as replenishment of work station \( (IVS_T < 0) \);

2) **Capital Structure Decision**—selecting the best portion of debt in the capital structure:
   Issuing New Debt Capital and Finding the value of \( \Theta \) which raises
   \[
   \text{VE}_0 = \text{discounted LFCFE}_t \text{'s at } ke \text{ as much as possible;}
   \]

3) **Capital Acquisition/Disbursement Decision**—New Equity Capital Issuance and Dividend Policy: how much \( \text{NEC}_t \) to issue and \( \text{NCDE}_t \) for each time period for max \( \text{VE}_0 \) taking Account of \( \Delta ke \) as \( \Theta \) changes: \( \text{VE}_0 = \text{present value of LFCFE}_t \text{'s discounted at } ke; \]

4) **Valuation of the entire Firm or its Equity portion: \text{VF}_0 \text{ or } \text{VE}_0**
   \[
   \text{VF}_0 = \text{present value of UFCFF}_t \text{'s and } TV_T \text{ discounted at } k_f^*;
   \]
   \[
   \text{VE}_0 = \text{present value of LFCFE}_t \text{'s and } TV_T \text{ discounted at } ke.
   \]

For each decision, the current \( t_0 \) cash flow and the \( \text{VE}_0 \) must be determined by discounting the future cash flows available to equity suppliers at the risk-adjusted cost of equity capital, or by discounting the differential net after-tax operating cash flows \( \Delta C_f^*_t \) at the risk-adjusted weighted-average cost of capital \( k_f^* \), or by discounting the UFCFF \( t \)’s at the risk-adjusted weighted-average cost of capital \( k_f^* \).

The future cash flows to be discounted must first be forecasted under all the assumptions of the proposed course of action and after taxes. Often, the future cash flows cannot be forecasted until after the future financial statements have been forecasted under the relevant assumptions. The capital structure \( \Theta = \frac{\text{VD}_0}{\text{VF}_0} \) must first be specified. Generally, the future pro-forma income statements are forecasted first, followed by the pro-forma balance sheets, followed by the pro-forma cash flow statements.

The first step in forecasting the pro-forma income statements is to forecast the growth rate of sales in each future year up to the Terminus Year and then the perpetual growth rate beyond the Terminus Year. The next step in forecasting the pro-forma income statements is to forecast the expenses and their dependence on sales in each year; the most common expense forecasting methods are: 1) fixed + variable expense; 2) percentage of sales with zero fixed expense. Often, regression analysis is used to identify the fixed and variable components of an expense as it relates to sales in past years, and then used to forecast the expense as a function of forecasted sales for each future year. In any case, a specific assumed relationship between each expense and future sales must be stated explicitly, with the foundation provided explicitly. Interest expense, of course, must depend on the amount of debt outstanding on the balance sheet, so the balance sheet and income statement are interconnected through the debt balance and interest expense. The increase in retained earnings comes from the net income after tax minus dividends paid, so this is another income statement-balance sheet interaction.
The discount rate used to compute the Net Present Value is the risk-adjusted required rate on all the capital employed in the course of action, the rate pertaining to the beneficiaries of the cash flows to be earned and the risk they perceive or bear. If the capital employed is a combination of debt and equity, the weighted average cost of capital, \( kf^* \), is used as the discount rate; if the capital employed is only equity, the cost of equity capital, \( ke \), is used as the discount rate. If we are discounting the forecasted net after-tax differential (incremental) cash flows produced by a proposed asset acquisition, 

\[
\Delta CF_t^* = IVS_t + \Delta NWC_t + TV_T + (\Delta S_t - \Delta C_t - \Delta D_t) (1 - \tau) + \Delta D_t,
\]

then we discount using 

\[
kf^* = \Theta kd (1 - \tau) + (1 - \Theta) ke.
\]

We generally assume that capital structure \( \Theta \) will not change during the lifetime of this proposed asset. We have three alternative models to estimate the risk-adjusted cost of equity capital:

1) \( ke = kd + \text{equity risk adjustment} \), from the “rule-of-thumb” model;

2) \( ke = \left[ \frac{d_i}{P_0} \right] + g_x \), from the Gordon model;

3) \( ke = R_f + \beta (E[R_M] - R_f) + \text{unique risk adjustment} \), from Capital Asset Pricing Model.

The larger the perceived risk borne by the capital supplier, the higher the discount rate he uses (and his agent the firm’s management must use) to discount forecasted future cash flows to present value. The larger the discount rate, the smaller the present value of the forecasted future cash flows.
FINANCING IMPLICATIONS DEDUCED FROM BALANCE SHEET FORECASTS

When forecasting the Balance Sheet, usually the left-hand-side is forecasted first, so that the Total Assets required for the forecasted future course of action are first determined; then the Total of Liabilities and Equity is equal to Total Assets, and the analyst moves up the right-hand-side of the Balance Sheet, leaving Notes Payable (short-term debt) as the Plug. **This method has the effect of deducing the amount of new capital required to finance and support the assets required to achieve the forecast.** (Recall from Sustainable Growth theory that to increase sales, either the operating efficiency of the assets must be increased—a very difficult task—or the quantity of assets must be increased; increasing the quantity of assets requires paying for the new assets; *i.e.*, financing them.) Making Notes Payable the Plug on the right-hand-side of the Balance Sheet deduces the quantity of new debt capital required to finance the implied new assets required to achieve the forecast, subject to the implicit assumptions regarding the operating efficiency of the assets.

If you deduce that Notes Payable is a positive algebraic amount on the right-hand-side of the Balance Sheet, that means that your firm must borrow new debt capital to finance the proposed actions and expected events by borrowing and using the new funds to purchase new capital assets and working capital, or to pay the development expenses. This is the general method used in Balance Sheet forecasting to determine the viability of a proposed future course of action: if the amount of new debt required is too large, then the proposed course of action will likely not succeed. Before forecasting the Balance Sheet with Notes Payable as the plug, it cannot be said how much new capital will be required to achieve the forecast activities and purchase the required assets: this is why forecasting only the Income Statement is never enough to see the future clearly. Forecasting the Income Statement without the Balance Sheet may tacitly assume a sharp and profound increase in operating efficiency which is not justified, or it may tacitly assume the firm can borrow whatever amount of debt it needs to finance the new assets required to achieve the forecasts, when in reality, no lender will commit such a quantity of funds because of the increased financial risk it will bear.

You may find that some forecasted values of Notes Payable are negative; *i.e.*, are algebraic negative amounts on the right-hand-side of the Balance Sheet. This implies that the other liabilities and equity amounts are too large for the assets forecasted, and Notes Payable must be negative to cause the Balance Sheet to Balance with the smaller amount of Total Assets. You can improve the Balance Sheet by adding the absolute value of your negative Notes Payable amount to the Cash account on the LHS and replacing the negative Notes Payable amount with a zero. Remember then to add the newly-created cash to the totals of Current Assets and Total Assets. This action will keep the Balance Sheet in Balance if you handle it correctly, but with a larger Balance Sheet total than before. Following this adjustment, you will then see that the forecasted course of action creates new cash from operations, suggesting that the forecasted course of actions will in fact be
successful. In the view of the Sustainable Growth Model, the firm will be creating cash available for disgorgement and will be growing at an actual rate smaller than the Sustainable-Rate-Assuming-No-Increase-in-Operating-Efficiency. The increase in the Sustainable Growth Rate will come about because of the new course of action assumed to be implemented, with new assets, a new business plan, newly improved operating efficiency, etc.
SUSTAINABLE GROWTH
(See Higgins Chapter 4; recall duPont Equation: \( \text{ROE} = \frac{\text{NI}}{\text{NW}} = \left(\frac{\text{S}}{\text{TA}}\right) \left(\frac{\text{NI}}{\text{S}}\right) \left(\frac{\text{TA}}{\text{NW}}\right) = \frac{\text{ROI}}{\left(1-\Theta\right)}; \text{ROI} = \frac{\text{NI}}{\text{TA}} \); recall Financial Cash Flows \( \text{TCC} = \text{IVS} \))

The sustainable growth rate of sales, \( g^* \), is the rate of growth of sales (S)
\[
g^* = \Delta \% \ \text{Sales} = \frac{(S_2 - S_1)}{S_1}
\]
which the firm can sustain in the ordinary course of events:
\[
g^* = \text{sustainable growth rate of sales} \equiv P \ R \ A \ T^\wedge.
\]

\( P = \) profit margin on sales = \( \frac{\text{NI}}{\text{S}} \)
\( R = \) retention rate = cash retention of firm/net income = \( \frac{\text{CRF}}{\text{NI}} = \left(\frac{\text{NI} - \text{Dividends}}{\text{NI}}\right) \)
\( A = \) total asset turnover = \( \frac{\text{S}}{\text{TA}_{\text{eoy}}} \)
\( T^\wedge = \) financial leverage increase this year = \( \frac{\text{TA}_{\text{eoy}}}{\text{NW}_{\text{bop}}} \)

"sustain" means “achieve and support with internally-generated funds without selling new common stock and holding capital structure constant.” Capital Structure = \( \frac{\text{VD}}{\text{VF}} = \frac{\text{TL}}{\text{TA}} = \Theta; \) also measured by \( \frac{\text{TA}}{\text{NW}} \).

The sustainable growth rate of sales is the growth rate in the value of equity; i.e., the rate of change in the value of equity per year; i.e., the return earned on equity divided by the beginning equity, achieved by operations and financing. The sustainable growth rate of sales is achieved by retaining cash from operations and investing that cash into new plant and equipment. The sustainable growth rate of sales achieved is equal to the cash retention of the firm from operations during the year as a portion of the beginning-of-period equity with which the firm began the year.
\[
g^* = \left(\frac{\text{NI}}{\text{S}}\right) \left(\frac{\text{CRF}}{\text{NI}}\right) \left(\frac{\text{S}}{\text{TA}_{\text{eoy}}}\right) \left(\frac{\text{TA}_{\text{eoy}}/\text{NW}_{\text{bop}}}{\text{CRF}/\text{NW}_{\text{bop}}}\right) = \frac{\text{CRF}}{\text{NW}_{\text{bop}}}
\]

\( g^* = \) Change in Equity/Beginning-of-Period Equity = \( (R) \ (\text{ROE}) = (R) \ (T) \ (\text{ROA}) = (R) \ (T) \ (P) \ (A) \)
\( \text{ROE} = \) Return on Equity (Net Worth) = \( \frac{\text{NI}}{\text{NW}} = \frac{\text{Net Income/Shareholders’ Equity}}{\text{TA}_{\text{eoy}}/\text{TA}_{\text{eoy}}} = \frac{\text{TA}_{\text{eoy}}/\text{NW}_{\text{bop}}}{\text{ROA} / (1-\Theta)} \)
\( \text{ROE} = \frac{\text{NI}}{\text{NW}} = \left(\frac{\text{NI}}{\text{S}}\right) \left(\frac{\text{S}}{\text{TA}}\right) \left(\frac{\text{TA}}{\text{NW}}\right) = \frac{\text{ROA}}{(1-\Theta)} \)

If the firm adds more capital to its balance sheet than the CRF, then it will be able to purchase more new fixed assets, and it will be able to grow at a faster rate than the sustainable growth rate. The firm will get this additional capital either from New Debt Capital or New Equity Capital, so we can say that the firm can achieve a faster growth rate than its sustainable growth rate by selling new common stock or borrowing more new debt, so that \( \text{IVS} = \text{TCC} = \text{CRF} + \text{NDC} + \text{NEC} > \text{CRF} \). The firm could also add more capital by increasing the retention rate this year compared with last year’s retention rate. This increase in the quantity of productive new plant and equipment creates the faster growth rate in sales than the firm could have sustained using only internally-generated-at-the-same-prior-rate funds: \( g > g^* \). We see that the firm has achieved a greater-than-sustainable growth rate by absorbing cash.

If the firm adds less capital to its balance sheet than the CRF, then it must either get rid of some of the cash from operations not paid out as dividends (by paying a “special dividend” or by repurchasing stock from the market, or purchasing non-productive assets like the condo in Vail, CO or the Rolls-Royce for the president). This payout of cash will allow the firm to purchase a smaller amount
of new productive plant and equipment than it otherwise would have, and its growth rate actually achieved will be smaller than \( g^* \), the rate it is capable of sustaining: \( g < g^* \). We see that the firm has grown at a slower rate than it could have sustained by disgorging cash.

If the firm adds exactly the amount of the retained cash \( CRF \) to its balance sheet as new plant and equipment, it will grow at exactly the sustainable growth rate: \( g = g^* \).

Management has only three levers for controlling ROE: (1) the earnings squeezed out of each dollar of sales, the profit margin on sales \( P = NI/S \); (2) the sales generated from each dollar of assets employed, the total assets turnover, \( A = S/TA \); (3) the amount of equity used to finance the assets, the financial leverage across time \( T^\wedge = TA/NW_{bop} \). (4) To convert the return on equity to an increase in the amount of equity we need to know how much of the cash provided by operations is retained and reinvested, \( R \). The amount which the firm can add to new assets and equity each year is the amount of cash flow from operations retained and reinvested and not paid out as dividends; \( R \) = the retention rate = Earnings Retained / Net Income After Tax = \( R = (NI - Dividends)/NI = 1 - Dividend Payout Rate d. \) The sustainable growth rate of sales \( g^* \) is the only growth rate in sales that is consistent with stable values of these four ratios \( P, R, A, \) and \( T^\wedge \). If sales increase at any other rate than \( g^* \), then at least one of these four ratios \( P, R, A, T^\wedge \), must change.

**Growth Faster than the Sustainable Rate**

\( g > g^* \): If the actual growth rate of sales \( g \) is greater than \( g^* \) this means that either: (1) the efficiency of operations must have improved (we see an increase in \( A=S/TA \) or an increase in \( P=NI/S \)), or (2) the quantity of assets used in operations has increased and financial policy has changed (we see an increase in \( R=(NI-Div)/NI \) or an increase in \( T^\wedge = TA/NW_{bop} \)). The company must either have: (1) increased operating efficiency or (2) absorbed more cash to purchase more assets. It is very difficult to increase operating efficiency, and we rarely see it in real firms. So most commonly, when we see \( g > g^* \) that rapid growth has been achieved by purchasing new assets, which required the absorption of new cash.

The company absorbs new cash from outside which it uses to purchase more capital assets which achieve the greater growth rate of sales, either from new borrowing (\( NDC = \) New Debt Capital) or newly sold common stock (\( NEC = \) New Equity Capital), or from retaining more cash from operations than it had previously (increasing \( CRF = \) Cash Retention of the Firm) which it does by reducing the payout ratio \( p \) as it increases the Retention Rate \( R = 1 - p \). Recall that \( NDC + NEC + CRF = TCC = IVS \).

If operations cannot become more efficient, then usually the company will absorb new cash by new borrowing, and we see \( NDC \) increase, and we usually see financial leverage \( \Theta \) increase as well. Firms that grow faster than their sustainable rate generally take in new cash through new borrowing. If they grow for many years at a rate faster than the sustainable rate, usually that means they have borrowed too much, and they fail when they can borrow no more. This is what Higgins means by “growing to disaster.”

To preclude failure, the firm must generate new equity by selling new common stock, or by retaining a larger portion of earnings each year. It might also have to increase selling prices, decrease production rates, or outsource some production activities. But it cannot continue to grow for many years at a rate exceeding its sustainable growth rate, without failing.
Growth Slower than the Sustainable Rate

If the actual growth rate of the firm \( g \) is smaller than its sustainable growth rate \( g^* \), the rate it could sustain if it wished, that must be because either: (1) a reduction has occurred in the operating efficiency of the firm’s assets (we see a reduction in \( A = S/TA \) or a reduction of \( P = NI/S \)), or (2) a reduction in the quantity of assets used in production, compared with the normal increase in new fixed assets, has occurred, or an opposite change in financial policy has occurred (we see a decrease in the retention rate \( R = (NI - Div)/NI \) with a concomitant increase in the payout ratio \( p \), or a decrease in the capital structure \( \Theta = TL/TA \) and financial leverage \( T^\gamma = TA/NW_{bop} \)). The growth rate is smaller than the sustainable rate because the firm has purchased new plant and equipment in a quantity smaller than is necessary to achieve its sustainable growth rate: \( IVS < CRF \). If the firm has reduced its purchases of new plant and equipment, it has unused cash available which it can pay out or disgorge. Generally, \( g < g^* \) means that the firm disgorges excess cash which is available to repay outstanding debt, so financial leverage \( \Theta \) can be reduced. A company growing at a rate smaller than its sustainable rate is often called a “cash cow” because it produces excess cash which an acquiring company can use for whatever it wishes.

The ordinary course of events is the following; notice the definitions of \( P, R, A, T^\gamma \):

1. The firm’s operating efficiency stays constant from year to year.
   Return on Investment or Assets = \( ROA = NI/TA = (S/TA) \times (NI/S) = \) constant
   \( A = \) Total asset turnover = \( S/TA = \) constant
   \( P = \) Profit margin on sales = return on sales = \( ROS = NI/Sales = \) constant.
   The constancy of operating efficiency means that the only way to make sales grow is to purchase more assets, because the firm cannot operate its existing assets so as to increase production.

2. The firm’s capital structure remains constant from year to year.
   \( \Theta = VD/VF = VD / (VD + VE) = \) constant
   \( T^\gamma = \) assets to equity leverage ratio = Assets/Equity = \( TA/NW_{bop} = \) constant
   We measure leverage using the end-of-period total assets divided by the beginning-of-period net worth.

   As a result of 1) and 2), the firm’s return on equity remains constant:
   \( ROE = NI/NW = (NI/TA) \times (TA/NW) = \) constant

3. The firm does not sell any new equity capital.
   \( NEC = 0 \)

4. The firm holds its "dividend policy" constant and its retention rate constant.
   \( R = \) retention rate, the portion of income retained and reinvested = \( (NI - Div)/NI \).
   The payout fraction, or payout ratio \( p = Dividends/NIAT = \) constant.
   This means that the firm holds its retention rate \( R = b = (NIAT - Dividends) / NIAT \) constant also, because \( b + p = R + p = 1.0 \).

5. To counteract the effect of increases in owners' equity resulting from the acceptance of positive-NPV projects, which would otherwise tend to reduce \( \Theta \), and the constancy of the retention rate, which also tends to reduce \( \Theta \), the firm borrows new debt capital only in the quantity necessary to hold \( \Theta \) constant.
\[ g^* = P \times R \times A \times T^\gamma = (NI/S) \times [(NI - Div)/NI] \times (S/TA) \times (TA/NW_{bop}) \]

By "sustain" we mean "support with funds likely to be generated in the normal course of business without selling new common stock". These funds consist of both the new debt capital borrowed plus the new equity capital generated from operations this year by positive-NPV projects, which is retained and reinvested. These funds will be used to purchase new assets to produce new units of product which will be sold, thereby creating the growth. That is, because we assume the operating efficiency to remain constant (i.e., \( NI/TA = (S/TA) \times (NI/S) = \) constant), new sales require new assets, and we must finance the assets (that is, pay for them) using new debt and new equity constrained in amount by the constancy of the capital structure ratio \( \Theta \) and the constant dividend payout ratio \( p \) and constant retention rate \( b \), and by the stipulation that the company will not sell new common stock; i.e., \( NEC = 0 \).

\[ \text{ROE} = \text{Return on Equity} = \text{Net Income} / \text{Shareholders' Equity} = \text{PAT} = \text{ROA}/(1 - \Theta) \]

Return on Equity = Profit Margin \times Asset Turnover \times Financial Leverage
\[ \text{ROE} = \text{NI/Equity} = \text{NI/Sales} \times \text{Sales/Assets} \times \text{Assets/Equity} \]

\[ \text{ROA} = \text{ROI} = \text{Return on Investment} = \text{Return on Assets} = \text{NIAT} / \text{Total Assets} \]

\[ P = \text{Profit Margin} = \text{Net Income}/\text{Sales} \]

\[ A = \text{Asset Turnover} = \text{Sales}/\text{Total Assets} \]

\[ T^\gamma = \text{Financial Leverage} = \text{Total Assets}/\text{Equity}_{bop} \]; that is, we use the beginning-of-period equity

Management has only three levers for controlling ROE: (1) the earnings squeezed out of each dollar of sales, the profit margin on sales \( P = \text{NI}/\text{Sales} \); (2) the sales generated from each dollar of assets employed, the total asset turnover \( A = \text{S}/\text{TA} \); (3) the amount of equity used to finance the assets, the financial leverage
\[ T^\gamma = \text{TA}/\text{NW}_{bop} \]. Management can affect the growth rate of sales and the growth rate of the return on equity by changing the retention rate \( R = (\text{NI} - \text{Div}) / \text{NI} \), the portion of income retained and reinvested (Gordon's \( b = 1 - p \)). The larger the retention rate, the faster growth will be because the greater will be the value of assets acquired to produce more product.

\[ R = \text{Retention Rate} = \text{Earnings Retained/Net Income After Tax} = (\text{NIAT} - \text{Dividends}) / \text{NIAT} \]

\[ R = b \] of the Gordon Model = \( 1 - d \)

\[ d = \text{payout ratio} = \text{Dividends} / \text{NIAT} \]

\[ d = \text{Dividend Payout Rate} = \text{Dividends/NIAT} = p = 1 - b \] of the Gordon Model, and \( p = 1 - R \).

In the Gordon Model, we also had \( r \) = average rate of return on new investment, the weighted average of the Internal Rates of Return of all investment projects accepted. Recall that in the Gordon model, the constant perpetual growth rate \( g_w = b \times r \), the retention rate multiplied by the rate of return on new investment.

Investment \( I_t \) at time \( t \) produces a perpetual stream of new income \( \Delta E_{t+1} \) beginning at \( t+1 \) and continuing forever. This new income \( \Delta E_{t+1} \) at \( t+1 \) is added to the previous year's income to get the new income:
\[ E_{t+1} = E_t + \Delta E_{t+1} \]. The amount of new income at \( t+1 \), \( E_{t+1} \), is equal to the product of the average rate of return on new investment, \( r \), multiplied by the amount of investment at time \( t \), \( I_t \):
\[ \Delta E_{t+1} = r \times I_t \].

\( g^* \) is the only growth rate in sales that is consistent with stable values of these four ratios, \( P \), \( R \), \( A \), and \( T \). If sales increase at any other rate, then at least one of these four ratios must change.
To achieve growth in sales $g$ faster than $g^*$ ($g > g^*$), the company must do at least one of the following:

1. increase its operating efficiency, either by improving the asset turnover $A$ and generating more sales, or improving its control of expenses and increasing the portion of sales earned as income $P$ and retaining this cash for investment;
2. increase its financial leverage by borrowing NDC at a faster rate and raising $T$;
3. retain more cash from operations by increasing the retention rate $R$ and reducing the payout ratio $p$;
4. sell new equity capital $NEC > 0$.

$g > g^*$ means that the company absorbs new cash from outside.

To achieve $g > g^*$ means that either operations must improve, or financial policy must change. The company generally must absorb new cash from outside to purchase the new assets required to increase sales. If operations cannot become more efficient, then financial leverage must increase or the firm must sell stock; firms that grow too fast must generally take in new cash through borrowing. Eventually, they can borrow no more, and they fail. To preclude failure, the firm must generate new equity by selling new common stock, or by retaining a larger portion of earnings each year. It might have to increase prices, decrease production rates, or outsource some production activities. But the firm cannot continue to grow for many years at a rate exceeding its sustainable growth rate, without failing.

To have growth in sales slower than $g^*$, the company must have at least one of the following conditions:

1. reduce its operating efficiency, either by reducing the asset turnover $A$ and generating fewer sales, or losing its control of expenses and reducing the portion of sales earned as income $P$, often by increasing salary payments or by purchasing non-operating assets such as skiing condominiums to be used by managers;
2. reduce its financial leverage by borrowing NDC at a slower rate and lowering $T$; or by paying off existing debt ahead of time and thereby reducing $T$ and $q$;
3. retain less cash from operations by reducing the retention rate $R$ and increasing the payout ratio $p$;
4. repurchase stock from the market: $NEC < 0$.

$g < g^*$ means that the firm disgorges cash: it generates excess cash which can be paid out as larger salaries, purchase of fun assets, repayments of debt, repurchases of stock, or larger dividends.
FINANCE 700 – SUMMER, 2008
WEEK 3: June 10 and 12, 2008
Case 1: Due June 10--SBW Prob. 10.16, a, b, c only; pp. 787-794 and 275-278. Wal-Mart
Below I provide a sample cover letter. Note the letterhead. Note the Inside Address. The financial analysis is spurious. You write a correct analysis.

J. STUART WOOD, Ph. D.
Consulting Economist
500 Arlington Drive
Metairie, Louisiana 70001
(504) 866-7200

January 29, 2008

Mr. Sam Walton
Chairman, Wal-Mart Stores, Inc.
702 S. W. 8th Street
Bentonville, Arkansas 72716

Re: Case 2: Problem 10.16, a,b,c,d: Pro-forma Statements of Wal-Mart Stores, Years 1 – 5

Dear Mr. Walton:

As you requested, I have forecasted pro-forma statements and ratio analysis factors of Wal-Mart Stores for Year 1 through Year 5, using the information presented in Stickney and the forecast assumptions on Stickney pages 788-793. I used these forecasted pro-forma statements as an Initial Scenario to analyze the interactive quality of the forecast assumptions made, and I adjusted the forecasted assumptions to remove the ($584 Million) Cash balance of Year 5, as requested in (c). I then analyzed the profitability and risk of Wal-Mart using financial ratios as requested in (d).

a, b: The basic assumptions on pages 788-793 caused the forecast of a negative cash balance at the end of year five, which I believe is not correct; this result is due to:_____________.

Also, there was an unacceptably low rate of return on common shareholders’ equity (NIAT/NW = ROE) for that future period due to sales growth at a rate smaller than the Sustainable Growth Rate and larger than the increases in the growth of Property, Plant, & Equipment, thereby generating cash which accumulated on the balance sheet and reduced the total assets turnover and hence the ROA; also, the capital structure ratio Total Liabilities /Total Assets increased as cash from operations increased net worth. Hence, alternative forecasts were prepared using an alternative Scenario assumption in an attempt to increase the ROE and generate cash on the balance sheet.

c. Debt was grown at a faster rate;

I found that Scenario c was able to produce a positive cash balance and an increase in the Return on Equity for each of the five future forecast years, compared with the initial forecast a, but the increase is smaller than was hoped: I recommend that Scenario c be implemented with larger buybacks of stock to reduce the value of equity, increase the financial leverage, and increase ROE still more than the initial Scenario a would. The initial Scenario a produced only very slight increases in ROE for each year, while Scenario c produced slight decreases in ROE. The alternative annual rates of return on common equity are shown in Table 1 in the body of the report. A different Scenario might produce superior risk-return performance.

Enclosed in the Appendix are two sets of pro-forma financial statements, cash flow statements, and (d) tables of ratios for the five-year period, one set for each Scenario a and c. Following an Executive Summary of the results of the two analysis Scenarios, I provide eight pages of detailed analysis of the forecasts and the alternative Scenarios.

It has been a pleasure providing this analysis for you. Please call if you have any questions or if there is other assistance I might render. Many thanks.

Sincerely,

J. Stuart Wood

Enclosure: Report and 16 Appendix Spreadsheets
**FREE CASH FLOWS TO VALUE THE FIRM AND TO VALUE EQUITY**

"Financial Free Cash Flows" Are Used to Value the Firm and to Value Equity.

The value of the firm is the present value to the owners of all the future benefits they will receive from the firm for the remainder of its lifetime forward from the present day.

Free Cash Flow (FCF) or Leveraged Free Cash Flow to Equity Suppliers (LFCFE) is the annual cash flow the firm can use to pay common dividends after the company has made all the investments in fixed assets and working capital necessary to sustain ongoing operations and without adversely affecting the planned growth of the firm; i.e., after purchasing plant and equipment needed to maintain operations and achieve growth, and after purchasing the net working capital required to operate the new plant and equipment. This definition of "Free Cash Flow" subtracts the required debt service (PP + I) for the year on the way to the computation of FCF. We call this "Leveraged Free Cash Flow to Equity Suppliers", LFCFE.

Another definition is: Free Cash Flow is the cash flow the firm can distribute to all investors after the company has made all the investments in fixed assets and working capital necessary to sustain ongoing operations and planned growth. This means that FCF is the cash which the firm can use to pay either stockholders (pay dividends or repurchase stock) or debtholders (PP or Interest) without adversely affecting the planned growth of the firm; i.e., after purchasing plant and equipment needed to maintain operations and achieve growth, and after purchasing the net working capital required to operate the new plant and equipment, but without having subtracted debt service. Stickney calls this "Unleveraged Free Cash Flow to the Firm", UFCFF.

Note that these two definitions are different, so you must specify what definition you are using.

**VALUE OF THE FIRM AND THE VALUE OF EQUITY**

The value of the firm is the present value to the owners of all the future benefits they will receive from the firm for the remainder of its lifetime forward from the present day. If we include "bondholders" in "owners", then the value of the debt is included in this value. If we do not include bondholders, then the owners are only the equity holders, and the value considers only the equity.

**VALUE OF THE FIRM**

\[ VF_0 = VF_0 = VD_0 + VE_0 \]

assuming the term-structure of interest rates is flat:

\[ VF_0 = \sum_{t=1}^{T} \left( \frac{UFCFF_t}{(1 + kf^*)^t} \right) + TV_T / (1 + kf^*)^T \]

\[ VF_0 = \sum_{t=1}^{T} \left( \frac{CAU_t - IVS_t}{(1 + kf^*)^t} \right) + TV_T / (1 + kf^*)^T \]

\[ VF_0 = \sum_{t=1}^{T} \left( \frac{CAC_t - IVS_t}{(1 + kf^*)^t} \right) + TV_T / (1 + kf^*)^T \]

\[ VF_0 = \sum_{t=1}^{T} \left( \frac{NCDD_t + NCDE_t}{(1 + kf^*)^t} \right) + TV_T / (1 + kf^*)^T \]

\[ VF_0 = \sum_{t=1}^{T} \left( \frac{NCDD_t + NCDE_t - ITS_t}{(1 + kf^*)^t} \right) + TV_T / (1 + kf^*)^T \]

Note that \( CAC > CAU \), so we must use a larger discount rate \( kf > kf^* \) to get the same present value when discounting CAC.

**VALUE OF EQUITY**

\[ VE_0 = VE_0 \text{ (assuming flat term-structure of interest rates):} \]

\[ VE_0 = VF_0 - VD_0 \]

\[ VE_0 = \sum_{t=1}^{T} \left( \frac{NCDE_t}{(1 + ke)^t} \right) + TV_T / (1 + ke)^T \]

\[ VE_0 = \sum_{t=1}^{T} \left( \frac{LFCFE_t}{(1 + ke)^t} \right) + TV_T / (1 + ke)^T \]

**TERMINAL VALUE** at time \( T = TV_T \):

\( TV_T = \text{Terminal Value}_T = \text{the value at a particular time point } T \text{ of all of the following cash flows for the remaining lifetime of the firm. The value of a future cash flow occurring at some future time point, valued at a particular time point } T, \text{ is computed by discounting that cash flow from its time point of occurrence back to the particular time point} \)

\[ TV_T = \]
\( T \) at the proper discount rate or "cost of capital". All of the present values at time \( T \) of all of the future cash flows are then added together to get \( TV_T \).

**NEW CONCEPTS TO RECONCILE ACCOUNTING INCOME WITH FINANCIAL CASH FLOWS:**

**Depreciation Tax Savings** (DTS) = \( \tau \) DEP = the income tax the firm does not have to pay because depreciation expense (a non-cash expense) is deductible from taxable income = \( \tau \) DEP, where \( \tau \) is the firm's total income tax rate and DEP is the depreciation expense for the year.

**Interest Tax Savings** (ITS) = \( \tau I \) = the income tax the firm does not have to pay because interest expense is tax-deductible = \( \tau I \), where \( \tau \) is the firm's income tax rate (Federal, State, and City) and \( I \) is this year's interest expense for all debt owed.

In valuing the firm, we neglect the distinction between accrued revenues and expenses and cash receipts and expenditures. In real life, of course, only the cash receipts are received and only the cash expenditures are paid. Analysis of past history of the firm must make this distinction; however, for forecasting the future, we generally assume no differences in the collection and payment rate across future time, so that we effectively ignore the increases in accounts receivable and in accounts payable. We recall that "operations" omits consideration of financing, so that "operating income" is the income prior to the subtraction of interest expense.

**NOPAT** = Net Operating Profit After Taxes is the operating profit the firm would have if it had no debt.

\[
\text{NOPAT} = \text{Net Operating Profit After Taxes} = \text{Cash Available to the Unlevered Firm} = \text{CAU}
\]

NOPAT is not a cash flow used in valuing the firm; it is merely used to compare firms with the same line of business but different capital structures, so as not to be misled by the differences in financial leverage between the firm. NOPAT is the net operating profit after tax. It is not related to the LFCFE or to the UFCFF, which are used to value the equity or to value the firm.

\[
\text{NOPAT} = \text{Net Operating Profit After Taxes} \quad \text{CAU}
\]

\[
\text{NOPAT} \quad \text{CAU}
\]

This hypothetical tax is larger than the actual tax the firm paid because the hypothetical tax is not reduced by the interest tax savings.

**CASH FLOW NOTATION:**

\( R_C = \) cash receipts from operations, whether accrued as this year's revenues or not;
\( E_X = \) cash expenditures for operating expenses, whether accrued for this year or not;

\[
\text{GCFO} = R_C - E_X = \text{Gross Cash from Operations is a hypothetical operating cash flow}
\]

If all revenues were collected and all expenses were paid, GCFO would be a real cash flow.

\[
\text{GCFO is the operating cash flow, neglecting depreciation (because it is not cash) and interest.}
\]

But income taxes, paid in cash, take account of both depreciation and interest, which reduce taxable income, so we must compute the cash income taxes paid correctly.

\[
\text{AITP} = \text{Actual Cash Income Taxes Paid} = \tau (\text{GCFO}) - \text{DTS} - \text{ITS}
\]

\[
\text{NOPAT} = \text{CAU} = \text{GCFO} - \text{AITP} - \text{ITS}
\]

Notice that when computing NOPAT = CAU, we ignore the reduction in income taxes which results from the interest expense the firm pays. NOPAT = CAU is a smaller number than if we reduced the taxes for the effect of interest deductibility.

\[
\text{NOPAT} = \text{CAU} = \text{GCFO} - \text{GTAX} + \text{DTS}
\]

NOPAT is the after-tax net income the firm would have if it had no debt.
To compute NOPAT, we do the following:

1. Subtract cash operating expenditures (*i.e.*, cash operating expenses omitting depreciation and other non-cash charges, and ignoring interest) from cash receipts: $RCPT - $EXPT

2. Subtract the hypothetical income tax the firm would pay if it had no debt and therefore had no tax-deductible interest expense. That means, however, that the tax-deductible operating non-cash expense *depreciation* does have its effect of lowering taxes recognized. However, the effect of the interest expense in lowering taxes is not considered in this calculation.

   This is equivalent to EBIT – Adjusted Taxes, where "adjusted taxes" means the larger income tax which would be paid in the absence of interest expense.

   If we take account of the interest tax savings, ITS, we add the interest tax savings to NOPAT = CAU, and we get the Cash Available to All Capital Suppliers = CAC.

$$CAC = CAU + ITS$$

The Cash Available to All Capital Suppliers is available to pay:

- I = interest expense on debt
- PP = principal payments on debt
- (CDD = Cash distribution to debt suppliers = I + PP)
- DIV = dividends on common stock and preferred stock
- TSTOCK = cash repurchase of common or preferred stock from the market.

$$CDE = \text{cash distribution to equity suppliers} = \text{DIV} + \text{TSTOCK}$$

If we subtract the investment outlays in new plant and equipment plus new net working capital

$$IVS = \text{New P&E} + \Delta NWC$$

From the CAC, we get the cash available to pay to all capital suppliers without adversely affecting the growth of the firm: $UFCFF = CAC - IVS$.

$$UFCFF = CAC - IVS$$

There is a slight discrepancy between the two methodologies, in that we say that $(CAC - IVS)$ is discounted at $kf$, while UFCFF is discounted at $kf^*$, depending on the context. I think this is unavoidable due to the vagaries of accounting, although I may be incorrect. Cope as best you can.

**CASH FLOWS:**

$$VF_t = VD_t + VE_t + \text{Value of Preferred Stock}_t$$

0. **Terminal Value** $TV_T$ = the value at a particular time point $T$ of all of the following cash flows for the remaining lifetime of the firm. The value of a future cash flow occurring at some future time point, valued at a particular time point $T$, is computed by discounting that cash flow from its time point of occurrence back to the particular time point $T$ at the proper discount rate or "cost of capital". All of the present values at time $T$ of all of the future cash flows are then added together to get $TV_T$.

$$TV_T = \text{Terminal Value at time } T = \frac{(\text{Relevant Cash Flow}_{T+1})}{(k - g_{\infty})}$$

where

- Cash Flow $T_{T+1} = \text{Cash Flow}_T \times (1 + g_{\infty})$
- $k = \text{the proper discount rate, either } ke \text{ or } kf^*$
- $g_{\infty} = \text{the constant perpetual growth rate from time } T \text{ to infinity}$.

The relevant cash flow is the same cash flow as was discounted explicitly at prior t's.
1. **LFCFE_t** = Leveraged Free Cash Flow to Equity at time \( t \)
   
   LFCFE_t is discounted at \( k_e \) to compute \( V_E_0 \).

   LFCFE_t can be used to pay common dividends or repurchase Treasury stock.

   \[
   \text{LFCFE}_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t - \text{IVS}_t - \Delta \text{NWC}_t - \text{PP}_t + \text{NDC}_t + \text{NPS}_t - \text{Pfd Divs}_t
   \]

   \( T = \) the total number of years specifically forecasted

   The interest expense + the tax effect of interest have already been subtracted from NIAT.

   Net New Debt Capital

   \[
   \text{NNDC}_t = \text{NDC}_t - \text{PP}_t
   \]

   Net cash distribution to equity

   \[
   \text{Dividends}_t - \text{New Equity Capital}_t
   \]

   ke = cost of equity capital = the rate of return required by the shareholders

   VF_0 = \sum_{t=1}^{T} \left[ \frac{\text{LFCFE}_t}{(1 + k_e)^t} \right] + TV_T / (1 + k_e)^T

   VE_0 = \sum_{t=1}^{T} \left[ \frac{\text{LFCFE}_t}{(1 + k_e)^t} \right] + TV_T / (1 + k_e)^T

   LFCFE subtracts changes in net working capital in defining LFCFE so that the only task for LFCFE is the ability to pay common dividends or repurchase Treasury stock. Neglecting preferred stock, we have:

   Leveraged Free Cash Flow to Equity \( t \)

   \[
   \text{LFCFE}_t = \text{NIAT}_t + \text{Noncash charges}_t - \text{IVS}_t - \Delta \text{NWC}_t + \Delta \text{NDC}_t + \text{NPS}_t - \text{Pfd Divs}_t
   \]

   where Net New Debt Capital = \( \text{NDC}_t - \text{PP}_t \)

   NWC = Net Working Capital = Current Assets - Current Liabilities

   \( \Delta \text{NWC} \) = the increase in Net Working Capital this year from that of last year

   NPS = New preferred stock

   Net cash distribution to equity = Dividends - New Equity Capital

   ke = cost of equity capital = the rate of return required by the shareholders

   VF_0 = \sum_{t=1}^{T} \left[ \frac{\text{LFCFE}_t}{(1 + k_f^*)^t} \right] + TV_T / (1 + k_f^*)^T

2. **UFCFF_t** = Unleveraged Free Cash Flow to the Firm at time \( t \) = UFCFF_t = Cash Flow from Operations before Subtracting Cash Outflows for Interest Costs (net of tax savings)_t - IVS_t - \( \Delta \text{NWC}_t \)

   UFCFF_t = \text{NIAT}_t + \text{Noncash charges}_t + \text{Interest Expense}_t (1 - \tau) - \text{IVS}_t - \Delta \text{NWC}_t

   UFCFF is discounted at \( k_f^* \) to give the value of the firm \( VF_0 = \text{VD}_0 + VE_0 \).

   UFCFF can be used to pay debt principal, pay debt interest, and pay dividends and repurchase common stock.

   UFCFF does not adversely affect the firm's planned growth because the investment outlays for new plant and equipment (IVS) and the working capital to support those outlays (\( \Delta \text{NWC} \)) have already been subtracted in its computation.

   VF_t = \text{VF}_t - \text{VD}_t - \text{Value of Preferred Stock}

   VF_0 = \sum_{t=1}^{T} \frac{\text{UFCFF}_t}{(1 + k_f^*)^t} + TV_T / (1 + k_f^*)^T

   UFCFF_t = \text{NIAT}_t + \text{DepExp}_t + \text{Int}_t (1 - \tau) - \text{IVS}_t

   kf* = weighted average cost of capital of the firm = \( \theta \) kd (1-\( \tau \)) + (1 - \( \theta \)) ke

   \( \theta \) = capital structure ratio of the firm = debt ratio of the firm = \( \text{VD} / \text{VF} \)
The firm should always minimize the cost of capital by reducing both business risk and financial risk.

NCE = Non-cash expenses; primarily depreciation and amortization.
"Unleveraged free cash flow from Operations" (UFCFO) is cash flow (that is, the non-cash expenses are added back to net income) before any payments to debtholders, either principal or interest (hence, it neglects financial leverage), but after income taxes (so it is "free" or "available to capital suppliers"). UFCFO = EBIT + Noncash expense – Income Tax

From that UFCFO, to get the UFCFF the unleveraged free cash flow to the firm, we must subtract the cash outlay necessary for new investment, IVS, and the new net working capital ΔNWC. UFCFF = UFCFO – IVS - ΔNWC.

So this cash flow begins with EBIT, adds back the noncash expenses, and then subtracts the tax which was paid:

NIATBI = net income after tax before interest = EBIT + NCE – Income Taxes.

Reported Cash Flow from Operations = NIAT + Noncash charges
Cash outflow for interest = Int
Interest Tax Savings = \(t\) Int
Interest cash outflow net of income-tax savings = Int - \(t\) Int = \((1 - \tau)\) Int

### Unleveraged Cash Flow from Operations

\[ \text{Unleveraged Cash Flow from Operations}_t = \text{UFCFO}_t = \]  
\[ \text{Reported Cash Flow from Ops}_t + \text{Int}_t (1-\tau) = \]  
\[ \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t + \text{Int}_t (1 - \tau) \]

### Unleveraged Free Cash Flow to the Firm

\[ \text{Unleveraged Free Cash Flow to the Firm}_t = \text{UFCFF}_t = \]  
\[ \text{UFCFO}_t - \text{IVS}_t - \Delta\text{NWC}_t = \]  
\[ \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t + \text{Int}_t (1 - \tau) - \text{IVS}_t - \Delta\text{NWC}_t \]

UFCFF can be used to pay debt principal, debt interest, dividends, and purchase common stock in the market.

**Free Cash Flows Used in Valuation of the Firm and of Equity—Memorize the following:**

0. **Terminal Value**  
\[ TV_T = \text{Terminal Value at time } T = (\text{Relevant Cash Flow}_{T+1}) / (k - g) \]

From the Gordon Constant-perpetual-growth model.  
Cash Flow \(T+1\) = Cash Flow \(T\) \(X\ (1 + g^\infty)\)  
k = the proper discount rate, either \(k_e\) or \(k_f^*\)

1. **LFCFE_t** = Leveraged Free Cash Flow to Equity at time t  
The stream of LFCFE_t’s is discounted at \(k_e\).  
LFCFE_t can be used to pay common dividends and buy new working capital

\[ \text{LFCFE}_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t - \text{IVS}_t - \text{PP}_t + \text{NDC}_t + \text{NPS}_t - \text{Preferred Dividends} \]

NPS = New preferred stock  
LFCFE can be used to pay dividends or invest in new working capital

\[ \text{VE}_0 = \sum_{t=1} T \left[ \frac{\text{NCDE}_t}{(1 + k_e)^t} \right] + \frac{TV_T}{(1 + k_e)^T} \]

Net cash distribution to equity, \(s\) = Dividends, \(s\) – New Equity Capital,  
\(k_e\) = cost of equity capital = the rate of return required by the shareholders
\[ V_E(t) = \sum_{t=1}^{T} \left\{ \frac{LFCFE_t}{(1 + ke)^t} \right\} + \frac{TV_T}{(1 + ke)^T} \]

Leveraged Free Cash Flow to Equity, 
= NIAT_t + Noncash charges_t - IVS_t + ΔNNDC_t

T = the number of years specifically forecasted

The interest expense + the tax effect of interest have already been subtracted from NIAT.

Net New Debt Capital, = NNDC_t = NDC_t - PP_t

\[ VF_0 = \sum_{t=1}^{T} \left\{ \frac{(CAU_t - IVS_t)}{(1 + k^*_f)^t} \right\} + \frac{TV_T}{(1 + k^*_f)^T} \]

VF_t = VD_t + VE_t + Value of Preferred Stock

2. UCFCC_t = Unleveraged Free Cash Flow to the Firm at time t = UCFCC_t = Cash Flow from Operations bef. Subtracting Cash Outflows for Interest Costs (net of tax savings)
= NIAT_t + Noncash charges_t + Interest Expense (1 - τ)

UCFCC is discounted at k^*_f.

UCFCC can be used to pay debt principal, pay debt interest, pay dividends, and purchase new working capital

\[ VF_t = VF_0 - VD_t - Value of Preferred Stock \]

\[ \frac{VF_t}{(1 + k^*_f)^t} \]

UCFCC_t = NIAT_t + DepExp_t + Int_t (1 - τ) - IVS_t

k^*_f = weighted average cost of capital of the firm = \( \theta \) kd (1 - τ) + (1 - \( \theta \)) ke

\( \theta = \) capital structure ratio of the firm = VD/VF

The firm should always minimize the cost of capital by reducing both business risk and financial risk.

NCE = Non-cash expenses; primarily depreciation and amortization.

"Unleveraged free cash flow from Operations" (UFCFO) is cash flow (that is, the non-cash expenses are added back to net income) before any payments to debtholders, either principal or interest (hence, it neglects financial leverage), but after income taxes (so it is "free" or "available to capital suppliers"). UCFFO = EBIT + Noncash expense - Income Tax

From that UFCFO, to get the UCFCC, the unleveraged free cash flow to the firm, we must subtract the cash outlay necessary for new investment, IVS: UCFCC = UFCFO - IVS.

So this cash flow begins with EBIT, adds back the noncash expenses, and then subtract the tax which was paid:

NIATBI_t = net income after tax before interest = EBIT + NCE - Income Taxes.

Reported Cash Flow from Operations = NIAT + Noncash charges
Cash outflow for interest = Int
Interest Tax Savings = τ Int
Interest cash outflow net of income-tax savings = Int - τ Int = (1 - τ) Int

Unleveraged Cash Flow from Operations_t = UFCFO_t =
Reported Cash Flow from Ops_t + Int_t (1-τ) =
NIAT_t + DEPR_t + AMORT_t + Int_t (1 - τ)

Unleveraged Free Cash Flow to the Firm_t = UCFCC_t =
UCFFO_t - IVS_t =
NIAT_t + DEPR_t + AMORT_t + Int_t (1 - τ) - IVS_t

UCFCC can be used to pay debt principal, debt interest, dividends, and purchase working capital.

3. An alternative definition of LFCFE also subtracts changes in net working capital in defining LFCFE so that the only task for LFCFE is the ability to pay common dividends. Neglecting preferred stock, we have:

\[ \text{Leveraged Free Cash Flow to Equity}_t = \text{LFCFE}_t = \]

= NIAT_t + Noncash charges_t - ΔNWCT - IVS_t + ΔNNDC_t

= ability to pay common dividends

where Net New Debt Capital_t = NNDC_t = NDC_t - PP_t
CASH FLOWS

1. **LFCFE** = Leveraged Free Cash Flow to Equity at time \( t \)

   LFCFE is discounted at \( ke \) to compute \( VE_0 \).

   LFCFE can be used to pay common dividends or repurchase Treasury stock.

   \[
   LFCFE = NIAT_1 + \text{Noncash charges}_1 - \text{IVS}_1 + \Delta\text{NDC}_t + \Delta\text{NWC}_t + \text{NPS}_t - \text{Pfd Divs}_t
   \]

   \( T = \) the number of years specifically forecasted

   The interest expense + the tax effect of interest have already been subtracted from NIAT.

   \[
   \text{Net New Debt Capital}_t = \text{NDC}_t - \text{PP}_t
   \]

   \[
   VE_0 = \sum_{t=1}^{T} \frac{\text{LFCFE}_t}{(1 + ke)^t} + TV_T / (1 + ke)^T
   \]

   LFCFE subtracts changes in net working capital in defining LFCFE so that the only task for LFCFE is the ability to pay common dividends or repurchase Treasury stock. Neglecting preferred stock, we have:

   [Leveraged Free Cash Flow to Equity, LFCFE]

   where Net New Debt Capital = \( \text{NDC}_t - \text{PP}_t \)

   - \( \text{NWC} = \text{Net Working Capital} = \text{Current Assets} - \text{Current Liabilities} \)
   - \( \Delta\text{NWC} = \) the increase in Net Working Capital this year from that of last year
   - \( \text{NPS} = \) New preferred stock

   \[
   VE_0 = \sum_{t=1}^{T} \frac{\text{NCDEt}}{(1 + ke)^t} + TV_T / (1 + ke)^T
   \]

2. **UFCFF** = Unleveraged Free Cash Flow to the Firm at time \( t = \) UFCFF

   Cash Flow from Operations before Subtracting Cash Outflows for Interest Costs (net of tax savings) – IVS - \( \Delta\text{NWC} \)

   \[
   \text{UFCFF} = \text{NIAT} + \text{Noncash charges} + \text{Interest Expense} (1 - \tau) - \text{IVS} - \Delta\text{NWC}
   \]

   UFCFF is discounted at \( kf* \) to give the value of the firm \( VF_0 = VD_0 + VE_0 \).

   UFCFF can be used to pay debt principal, pay debt interest, and pay dividends and repurchase common stock.

   UFCFF does not adversely affect the firm’s planned growth because the investment outlays for new plant and equipment (IVS) and the working capital to support those outlays (\( \Delta\text{NWC} \)) have already been subtracted in its computation.

   \[
   VF_0 = \sum_{t=1}^{T} \frac{\text{UFCFF}_t}{(1 + kf*)^t} + TV_T / (1 + kf*)^T
   \]

   \( kf* = \) weighted average cost of capital of the firm = \( \theta kd (1 - \tau) + (1 - \theta) ke \)

   \( \theta = \) capital structure ratio of the firm = debt ratio of the firm = \( \frac{VD}{VF} \)

   The firm should always minimize the cost of capital by reducing both business risk and financial risk.

   See Stickney page 749:
NCE = Non-cash expenses; primarily depreciation and amortization.

"Unleveraged free cash flow from Operations" (UFCFO) is cash flow (that is, the non-cash expenses are added back to net income) before any payments to debtholders, either principal or interest (hence, it neglects financial leverage), but after income taxes (so it is "free" or "available to capital suppliers"). UFCFO = EBIT + Noncash expense – Income Tax

From that UFCFO, to get the UFCFF the unleveraged free cash flow to the firm, we must subtract the cash outlay necessary for new investment, IVS, and the new net working capital ΔNWC. UFCFF = UFCFO – IVS - ΔNWC.

So this cash flow begins with EBIT, adds back the noncash expenses, and then subtracts the tax which was paid:

NIATBI = net income after tax before interest = EBIT + NCE – Income Taxes.

Reported Cash Flow from Operations = NIAT + Noncash charges
Cash outflow for interest = Int
Interest Tax Savings = τ Int
Interest cash outflow net of income-tax savings = Int - τ Int = (1 - τ) Int

\[
\text{Unleveraged Cash Flow from Operations} = \text{UFCFO} = \text{Reported Cash Flow from Ops} + \text{Int} (1-\tau) = \text{NIAT} + \text{DEPR} + \text{AMORT} + \text{Int} (1 - \tau)
\]

\[
\text{Unleveraged Free Cash Flow to the Firm} = \text{UFCFF} = \text{UFCFO} - \text{IVS} - \Delta\text{NWC} = \text{NIAT} + \text{DEPR} + \text{AMORT} + \text{Int} (1 - \tau) - \text{IVS} - \Delta\text{NWC}
\]

UFCFF can be used to pay debt principal, debt interest, dividends, and purchase common stock in the market.

FREE CASH FLOWS USED IN VALUATION OF THE FIRM AND OF EQUITY:

0. Terminal Value \( T = TV_T = \text{the value at a particular time point} \ T \text{ of all of the following cash flows for the remaining lifetime of the firm. The value of a future cash flow occurring at some future time point, valued at a particular time point} \ T, \text{ is computed by discounting that cash flow from its time point of occurrence back to the particular time point} \ T \text{ at the proper discount rate or "cost of capital". All of the present values at time} \ T \text{ of all of the future cash flows are then added together to get} \ TV_T. \)

\[
TV_T = \text{Terminal Value at time} \ T = \left( \frac{\text{Relevant Cash Flow}_{T+1}}{k - g} \right) / (1 + k)^T
\]

From the Gordon Constant-perpetual-growth model.
Cash Flow \( T+1 = \text{Cash Flow}_T \times (1 + g) \)
\( k = \text{the proper discount rate, either} \ ke \text{or} \ kf^* \)
Remember to discount the Terminal Value back to Present Value by dividing it by \((1+k)^T\)

1. \( \text{LFCFE}_t = \text{Leveraged Free Cash Flow to Equity at time} \ t \)
\( \text{LFCFE} \text{ is discounted at} \ ke. \)
\( \text{LFCFE} \text{ can be used to pay common dividends and buy new working capital} \)

\[
\text{LFCFE} = \text{NIAT} + \text{DEPR} + \text{AMORT} - \text{IVS} - \text{PP} + \text{NDC} + \text{NPS} - \text{Preferred Dividends}
\]

\[
\text{NPS} = \text{New preferred stock}
\text{LFCFE} \text{ can be used to pay dividends or invest in new working capital}
\]

\[
\text{VE}_0 = \text{((t=1 to T)Σ NCDEt} / (1 + ke)^t + TV_T / (1 + ke)^T)
\]
Net cash distribution to equity \( t = \text{Dividends}_t - \text{New Equity Capital}_t \)
\( ke = \text{cost of equity capital} = \text{the rate of return required by the shareholders} \)

\[
\text{VE}_0 = \text{((t=1 to T)Σ LFCFE}_t / (1 + ke)^t + TV_T / (1 + ke)^T)
\]
Leveraged Free Cash Flow to Equity \( t = \)
Finance 700, Summer, 2008, Dr. Stuart Wood, Syllabus  Page 42 of 91 Pages

2. \( \text{UFCFF}_t = \text{Unleveraged Free Cash Flow to the Firm at time } t = \) 
Cash Flow from Operations bef. Subtracting Cash Outflows for Interest Costs (net of tax savings)  
= NIAT + Noncash charges + Interest Expense (1 - \( \tau \))  
UFCFF is discounted at \( \text{kf}^* \).  
UFCFF can be used to pay debt principal, pay debt interest, pay dividends, and purchase new working capital.  
\( \text{VE}_t = \text{VF}_t - \text{VD}_t = \text{Value of Preferred Stock} \)  
\( \text{VF}_t = (\text{t}=1 \text{ to } T) \sum (\text{CAU}_t - \text{IVS}_t) / (1 + \text{kf}^*)^t + \text{TV}_T / (1 + \text{kf}^*)^T \)  
\( \text{UFCFF}_t = \text{NIAT}_t + \text{DepExp}_t + \text{Int}_t (1 - \tau) - \text{IVS}_t \)  
\( \text{kf}^* = \text{weighted average cost of capital of the firm} = \theta \text{kd} (1-\tau) + (1 - \theta) \text{ke} \)  
The firm should always minimize the cost of capital by reducing both business risk and financial risk.  

NCE = Non-cash expenses; primarily depreciation and amortization.  
"Unleveraged free cash flow from Operations" (UFCFO) is cash flow (that is, the non-cash expenses are added back to net income) before any payments to debtholders, either principal or interest (hence, it neglects financial leverage), but after income taxes (so it is "free" or "available to capital suppliers"). UFCFO = EBIT + Noncash expense – Income Tax  
From that UFCFO, to get the UFCFF the unleveraged free cash flow to the firm, we must subtract the cash outlay necessary for new investment, IVS. UFCFF = UFCFO - IVS.  
So this cash flow begins with EBIT, adds back the noncash expenses, and then subtracts the tax which was paid: NIATBI = net income after tax before interest = EBIT + NCE – Income Taxes.  

\begin{align*}  
\text{Reported Cash Flow from Operations} & = \text{NIAT} + \text{Noncash charges} \times (1 - \tau) \\
\text{Interest Tax Savings} & = \text{Int} \times (1 - \tau) \text{Int} = (1 - \tau) \text{Int} \\
\end{align*}  

Unleveraged Cash Flow from Operations = UFCFO =  
\( \text{Reported Cash Flow from Ops} + \text{Int} (1-\tau) = \text{NIAT} + \text{DEPR} + \text{AMORT} + \text{Int} (1 - \tau) \)  

Unleveraged Free Cash Flow to the Firm = UFCFF =  
\( \text{UFCFO} - \text{IVS} = \)  
\( \text{NIAT} + \text{DEPR} + \text{AMORT} + \text{Int} (1 - \tau) - \text{IVS} \)  

UFCFF can be used to pay debt principal, debt interest, dividends, and purchase working capital.  

3. \( \text{LFCFE}_t = \text{Leveraged Free Cash Flow to Equity}_t \)  
An alternative definition of LFCFE also subtracts changes in net working capital in defining LFCFE so that the only task for LFCFE is the ability to pay common dividends. Neglecting preferred stock, we have:  
\begin{align*}  
\text{Leveraged Free Cash Flow to Equity}_t & = \text{LFCFE}_t =  \\
& = \text{NIAT}_t + \text{Noncash charges}_t - \Delta \text{NWC}_t - \text{IVS}_t + \Delta \text{NNDC}_t  \\
& = \text{ability to pay common dividends} \\
\end{align*}  

where Net New Debt Capital\(_t\) = NNDC\(_t\) = NDC\(_t\) – PP\(_t\)  

DEFINITIONS AND EQUATIONS FOR VALUATION:  
Financial Free Cash Flows, UFCFF, LFCFE Are Used to Value the Firm and to Value Equity.  
\( \text{ke} = \text{cost of equity capital} = \text{the risk-adjusted rate of return required by the shareholders} \)  
\( \text{ke} = \text{kd} + \text{risk adjustment} = d_t / P_0 + g_w = \text{R}_t + \beta (E[R_M] - R_f) \)  
\( \text{kf}^* = \text{the weighted-average cost of capital} = \theta \text{kd} (1-\tau) + (1 - \theta) \text{ke} \)
LFCFE_t = NIAT_t + DEPR_t + AMORT_t - IVS_t - ΔNWC_t - PP_t + NEC_t + NDC_t + NPS_t - Pfd Divs_t

UFCFF_t = NIAT_t + DEPR_t + AMORT_t + Int_t (1 - τ) - IVS_t + ΔNWC_t + NEC_t + NDC_t + NPS_t - Pfd Divs_t

ΔNWC_t = NWC_t - NWC_{t-1}, and NWC_t = CA_t - CL_t

The value of the firm \( VF_0 \) is the present value to the owners of all the future benefits they will receive from the firm for the remainder of its lifetime forward from the present day. If we include "bondholders" in "owners", then the value of the debt is included in this value. If we do not include bondholders, then the owners are only the equity holders, and the value considers only the equity, so it is called Value of Equity, \( VE_0 \).

The value of the firm \( VF_0 \) is the present value to the owners (debt holders and equity holders) of all the future benefits they will receive from the firm for the remainder of its lifetime forward from the present day. The value of equity \( VE_0 \) is the value of the firm minus the value of debt: \( VE_0 = VF_0 - VD_0 \). The value of equity \( VE_0 \) is also the present value to the equity holders of all the future net benefits (dividends minus new equity capital supplied) they will receive from the firm for the remainder of its lifetime forward from the present day.

\[
VF_0 = \sum_{t=1}^{T} \left( \frac{LFCFE_t}{(1 + ke)^t} + TV_T / (1 + ke)^T \right)
\]

\[
VE_0 = VF_0 - VD_0
\]

\[
TV_T = \text{Terminal Value at time } T = \frac{(LFCFE_{T+1})}{(ke - g_T)}
\]

Free Cash Flow (FCF) or "Free Cash Flow to Equity" (FCFE) or "Leveraged Free Cash Flow to Equity" (LFCFE) is the annual cash flow the firm can use to pay common dividends after the company has made all the investments in fixed assets and working capital necessary to sustain ongoing operations and without adversely affecting the planned growth of the firm; i.e., after purchasing plant and equipment needed to maintain operations and achieve growth, and after purchasing the net working capital required to operate the new plant and equipment. This definition of "Free Cash Flow" subtracts the required debt service (PP + 1) for the year on the way to the computation of FCF. We call this "Leveraged Free Cash Flow to Equity Suppliers", LFCFE. (Stickney)

\[
VE_0 = \sum_{t=1}^{T} \left( \frac{(LFCFE_t)}{(1 + ke)^t} + TV_T / (1 + ke)^T \right)
\]

TERMINAL VALUE at time \( T = TV_T \):

\[
TV_T = \text{Terminal Value at time } T = \frac{(Relevant \ Cash \ Flow_{T+1})}{(ke - g_T)}
\]

COST OF CAPITAL, THE DISCOUNT RATE FOR FUTURE CASH FLOWS:

1. Weighted-average cost of capital, \( kf* = \theta kd (1 - \tau) + (1 - \theta) ke \)

Used to discount UFCFF's.

2. Cost of Equity Capital,
\[ ke = Rf + \beta (E[RM] - Rf) + \text{non-systematic risk factors} \]

Used to discount LFCFE's.

\[ ke \] is determined by the Capital Asset Pricing Model:
\[ ke = Rf + \beta (E[RM] - Rf). \]

If another risk adjustment is necessary, it must be added.

The value of the firm \( VF_0 \) is the present value to the owners (debt holders and equity holders) of all the future benefits they will receive from the firm for the remainder of its lifetime forward from the present day. The value of equity \( VE_0 \) is the value of the firm minus the value of debt: \( VE_0 = VF_0 - VD_0 \). The value of equity \( VE_0 \) is also the present value to the equity holders of all the future net benefits (dividends minus new equity capital supplied) they will receive from the firm for the remainder of its lifetime forward from the present day.

We calculate the Value of Equity \( VE_0 \) by discounting the series of the "Leveraged Free Cash Flows to Equity at each time point" (LFCFE\(_t\)) from \( t=1 \) through \( t=T \) plus the Terminal Value at \( T \), all by the cost of equity capital \( ke \).

Free Cash Flow (FCF) or "Free Cash Flow to Equity" (FCFE) or "Leveraged Free Cash Flow to Equity" (LFCFE) is the annual cash flow the firm can use to pay common dividends after the company has made all the investments in fixed assets and working capital necessary to sustain ongoing operations and without adversely affecting the planned growth of the firm; i.e., after purchasing plant and equipment needed to maintain operations and achieve growth, and after purchasing the net working capital required to operate the new plant and equipment. This definition of "Free Cash Flow" subtracts the required debt service (\( PP + I \)) for the year on the way to the computation of FCF. We call this "Leveraged Free Cash Flow to Equity Suppliers", LFCFE. (Stickney)

\[
VE_0 = \sum_{t=1}^{T} \left[ \frac{(LFCFE_t)}{(1 + ke)^t} \right] + TV_T / (1 + ke)^T
\]

**TERMINAL VALUE** at time \( T = TV_T \):
\[ TV_T = \text{Terminal Value at time } T = \text{the value at a particular time point } T \text{ of all of the following cash flows for the remaining lifetime of the firm; } i.e., \text{beginning at } T+1 \text{ and going on to } \infty. \] The value of a future cash flow occurring at some future time point, valued at a particular time point \( T \), is computed by discounting that cash flow from its time point of occurrence back to the particular time point \( T \) at the proper discount rate or "cost of capital". All of the present values at time \( T \) of all of the future cash flows are then added together to get \( TV_T \).

From the Gordon Constant-perpetual-growth model,

\[
TV_T = \frac{\text{Relevant Cash Flow}_{T+1}}{(ke - g_m)}
\]

where

- Cash Flow \( _{T+1} = \) Cash Flow \( _T \times (1 + g_m) \)
- \( k \) = the proper discount rate, \( ke \) from the Capital Asset Pricing Model, usually;
- \( g_m \) = the constant perpetual growth rate from time \( T \) to infinity.

The relevant cash flow is the same cash flow as was discounted explicitly at prior \( t \)'s.

\( LFCFE_t = \) Leveraged Free Cash Flow to Equity at time \( t \)

The stream of \( LFCFE_t \)'s plus \( TV_T \) is discounted at \( ke \) to compute \( VE_0 \).

LFCFE can be used to pay common dividends or repurchase Treasury stock.

In any year \( t \),

\[
LFCFE_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t - \text{IVS}_t - \Delta\text{NWC}_t - \text{PP}_t + \text{NDC}_t + \text{NPS}_t - \text{Pfd Divs}_t
\]

LFCFE \( t \) = Leveraged Free Cash Flow to Equity, \( t \) = available dividends in year \( t \) =
\[ = \text{NIAT}_t + \text{Noncash charges}_t - \text{IVS}_t - \Delta\text{NWC}_t + \Delta\text{NDC}_t + \text{NPS}_t - \text{Pfd Divs}_t \]

\( T \) = the number of years specifically forecasted; in Problem 11.4, \( T = 17 \).

The interest expense + the tax effect of interest have already been subtracted from NIAT.

Net New Debt Capital \( = \text{NNDC}_t = \text{NDC}_t - \text{PP}_t \)

LFCFE subtracts changes in net working capital in defining LFCFE so that the only task for LFCFE is the ability to pay common dividends (or repurchase Treasury stock). LFCFE subtracts changes in net working capital in
defining LFCFE so that the only task for LFCFE is the ability to pay common dividends (or repurchase Treasury stock).

Neglecting preferred stock, we have:

Leveraged Free Cash Flow to Equity, \( LFCFE_t = \)

\[ LFCFE_t = NIAT_t + \text{Noncash charges}_t - \Delta NWC_t - IVS_t + \Delta NNDC_t \]

where Net New Debt Capital \( = NNDC_t = NDC_t - PP_t \)

\( NWC = \) Net Working Capital = Current Assets – Current Liabilities

\( \Delta NWC = \) the increase in Net Working Capital this year from that of last year

\( NPS = \) New preferred stock

\[
VE_0 = \sum_{t=1}^{T} \left[ \frac{LFCFE_t}{1 + ke} \right]^t + TV_T / (1 + ke)^T
\]

The value of the firm \( VF_0 \) is the present value to the owners of all the future benefits they will receive from the firm for the remainder of its lifetime forward from the present day. If we include "bondholders" in "owners", then the value of the debt is included in this value. If we do not include bondholders, then the owners are only the equity holders, and the value considers only the equity.

VALUE OF THE FIRM = \( VF_0 = VD_0 + VE_0 \) (assuming flat term-structure of interest rates):

\[
VF_0 = \sum_{t=1}^{T} \left[ \frac{UFCFF_t}{1 + kf^*} \right]^t + TV_T / (1 + kf^*)^T
\]

VALUE OF EQUITY = \( VE_0 \) (assuming flat term-structure of interest rates):

\( VE_0 = VF_0 - VD_0 \)

\[
VF_t = VD_t + VE_t + \text{Value of Preferred Stock}_t
\]

0. Terminal Value \( TV_T = \) the value at a particular time point \( T \) of all of the following cash flows for the remaining lifetime of the firm. The value of a future cash flow occurring at some future time point, valued at a particular time point \( T \), is computed by discounting that cash flow from its time point of occurrence back to the particular time point \( T \) at the proper discount rate or "cost of capital". All of the present values at time \( T \) of all of the future cash flows are then added together to get \( TV_T \).

\[
TV_T = \frac{\text{Terminal Value at time } T = (\text{Relevant Cash Flow}_{T+1})}{(k - g_m)}
\]

From the Gordon Constant-perpetual-growth model,

where

\[
\text{Cash Flow}_{T+1} = \text{Cash Flow}_T \times (1 + g_m)
\]

\( k = \) the proper discount rate, either \( ke \) or \( kf^* \)

\( g_m = \) the constant perpetual growth rate from time \( T \) to infinity.

The relevant cash flow is the same cash flow as was discounted explicitly at prior \( t \)'s.

2. \( UFCFF_t = \) Unleveraged Free Cash Flow to the Firm at time \( t = UFCFF_t = \)

Cash Flow from Operations before Subtracting Cash Outflows for Interest Costs (net of tax savings) – IVS - \( \Delta NWC + NDC_t \)

\[ UFCFF_t = NIAT_t + \text{Noncash charges}_t + \text{Interest Expense}_t (1 - \tau) - IVS_t - \Delta NWC_t + NDC_t \]

\[ UFCFF_t = NIAT_t + DEPR_t + \text{AMORT}_t + \text{Interest Expense}_t (1 - \tau) - IVS_t - \Delta NWC_t + NDC_t \]

\( UFCFF \) is discounted at \( kf^* \) to give the value of the firm \( VF_0 = VD_0 + VE_0 \).

\( UFCFF \) can be used to pay debt principal, pay debt interest, and pay dividends and repurchase common stock.

\( UFCFF \) does not adversely affect the firm's planned growth because the investment outlays for new plant and equipment (IVS\(_t\)) and the working capital to support those outlays (\( \Delta NWC\(_t\)\)) have already been subtracted in its computation.

\[ VE_t = VF_t - VD_t - \text{Value of Preferred Stock} \]
VF(t) = (t=1 to T) \sum \text{UFCFF}_t / (1 + k^*)^t + TV_T / (1 + k^*)^T
\text{UFCFF}_t = \text{NIAT}_t + \text{DepExp}_t + \text{Int}_t(1 - \tau) - \text{IVS}_t
k^* = \text{weighted average cost of capital of the firm} = \theta \text{kd}(1 - \tau) + (1 - \theta) \text{ke}
\theta = \text{capital structure ratio of the firm} = \text{debt ratio of the firm} = \text{VD/VF}

The firm should always minimize the cost of capital by reducing both business risk and financial risk.

See Stickney page 817:

NCE = Non-cash expenses; primarily depreciation and amortization.

"Unleveraged free cash flow from Operations" (UFCFO) is cash flow (that is, the non-cash expenses are added back to net income) before any payments to debtholders, either principal or interest (hence, it neglects financial leverage), but after income taxes (so it is "free" or "available to capital suppliers"). UFCFO = EBIT + Noncash expense – Income Tax
From that UFCFO, to get the UFCFF the unleveraged free cash flow to the firm, we must subtract the cash outlay necessary for new investment, IVS, and the new net working capital \( \Delta \text{NWC} \). UFCFF = UFCFO – IVS - \( \Delta \text{NWC} \).
So this cash flow begins with EBIT, adds back the noncash expenses, and then subtracts the tax which was paid:

\[ \text{NIATBI} = \text{net income after tax before interest} = \text{EBIT} + \text{NCE} - \text{Income Taxes}. \]
Reported Cash Flow from Operations = NIAT + Noncash charges
Cash outflow for interest = Int
Interest Tax Savings = \( \tau \text{Int} \)
Interest cash outflow net of income-tax savings = \text{Int} - \( \tau \text{Int} = (1 - \tau) \text{Int} \)

Unleveraged Cash Flow from Operations
\[ \text{UFCFO}_t = \text{Reported Cash Flow from Ops}_t + \text{Int}_t(1 - \tau) = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t + \text{Int}_t(1 - \tau) + \text{NDC}_t \]

Unleveraged Free Cash Flow to the Firm
\[ \text{UFCFF}_t = \text{UFCFO}_t - \text{IVS}_t - \Delta \text{NWC}_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t + \text{Int}_t (1 - \tau) - \text{IVS}_t - \Delta \text{NWC}_t + \text{NDC}_t \]

UFCFF can be used to pay debt principal, debt interest, dividends, and purchase common stock in the market. This is, in my view, the more useful understanding of UFCFF.

(An alternative definition of UFCFF does not subtract the change in Net Working Capital so that the UFCFF can also be used to purchase new working capital. I think this is not a helpful way of looking at things, because the new working capital is needed for the planned investment outlays in new plant and equipment. But you may sometimes see this.
(Alt 2. UFCFF = Unleveraged Free Cash Flow to the Firm at time t = UFCFF =
Cash flow from Operations bef. Subtracting Cash Outflows for Interest Costs (net of tax savings)
( = NIAT + Noncash charges + Interest Expense (1 - \tau) + NDC_t
UFCFF is discounted at k^*.
( this definition of UFCFF can be used to pay debt principal, pay debt interest, pay dividends, and purchase new working capital.)
\[ \text{VE}_t = \text{VF}_t - \text{VD}_t - \text{Value of Preferred Stock} \]
\[ \text{VF}_0 = (t=1 to T) \sum \text{UFCFF}_t / (1 + k^*)^t + TV_T / (1 + k^*)^T \]
\text{UFCFF}_t = \text{NIAT}_t + \text{DepExp}_t + \text{Int}_t(1 - \tau) - \text{IVS}_t
k^* = \text{weighted average cost of capital of the firm} = \theta \text{kd}(1 - \tau) + (1 - \theta) \text{ke}
\theta = \text{capital structure ratio of the firm} = \text{debt ratio of the firm} = \text{VD/VF}
The firm should always minimize the cost of capital by reducing both business risk and financial risk.
NCE = Non-cash expenses; primarily depreciation and amortization.
"Unleveraged free cash flow from Operations" (UFCFO) is cash flow (that is, the non-cash expenses are added back to net income) before any payments to debtholders, either principal or interest (hence, it neglects financial leverage), but after income taxes (so it is "free" or "available to capital suppliers"). UFCFO = EBIT + Noncash expense – Income Tax
From that UFCFO, to get the UFCFF the unleveraged free cash flow to the firm, we must subtract the cash outlay necessary for new investment, IVS. UFCFF = UFCFO - IVS. This alternative definition of UFCFF neglects the needed additional investment in new Net Working Capital to support the new investments.
So this cash flow begins with EBIT, adds back the noncash expenses, and then subtracts the tax which was paid:

\[ \text{NIATBI} = \text{net income after tax before interest} = \text{EBIT} + \text{NCE} - \text{Income Taxes}. \]

Reported Cash Flow from Operations = NIAT + Noncash charges
Cash outflow for interest = Int
Interest Tax Savings = \( \tau \) Int
Interest cash outflow net of income-tax savings = Int - \( \tau \) Int = \( (1 - \tau) \) Int

**Alternative Unleveraged Cash Flow from Operations**
\[ \text{UFCFO}_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t + \text{Int}_t \left(1 - \tau\right) + \text{NDC}_t; \text{ note no } -\Delta\text{NWC} \]

**Alternative Unleveraged Free Cash Flow to the Firm**
\[ \text{UFCFF}_t = \text{UFCFO}_t - \text{IVS}_t + \text{NDC}_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t + \text{Int}_t \left(1 - \tau\right) + \text{NDC}_t - \text{IVS}_t; \text{ note no } -\Delta\text{NWC} \]

This alternative UFCFF can be used to pay debt principal, debt interest, dividends, and purchase working capital to support the new investments.

Discuss any differences between your two VE computations. Compare your computed values with the actual market value of the equity of the firm and the total market value of the firm, and with the book value of the total assets and with the book value of equity. Explain any differences. Discuss your valuation procedures and what you have learned.

Cost of Capital: the rate of return required to be earned by the capital suppliers to compensate for bearing the risk they perceive.

**CAPM:**
\[ \text{ke} = R_f + \beta \left(\text{E}[R_M] - R_f\right) + \varphi \]

We use the cost of equity capital to discount cash flows received by equity suppliers, such as dividends, or the LFCFE’s.

**LFCFE_t = NIAT_t + DEPR_t + AMORT_t - IVS_t - \Delta\text{NWC}_t - \text{PP}_t + \text{NDC}_t + \text{NPS}_t - \text{Pfd Divs}_t**

**WACC:**
\[ \text{kf*} = \Theta \text{kd} \left(1 - \tau\right) + (1 - \Theta) \text{ke} \]

We use the weighted-average cost of capital to discount cash flows received by all capital suppliers, debt suppliers and equity suppliers, such as differential net after-tax operating cash flows generated by an investment project, or the UFCFF’s.

**Unleveraged Free Cash Flow to the Firm**
\[ \text{UFCFF}_t = \text{UFCFO}_t - \text{IVS}_t - \Delta\text{NWC}_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t + \text{Int}_t \left(1 - \tau\right) - \text{IVS}_t - \Delta\text{NWC}_t \]

Measurement of Economic Value Added and Economic Profit. We all want “superior performance”; but superior to what? Capital should be invested in the most profitable enterprise; we must take into account the alternative use for the capital which we used here. The *cost of capital* kf* is the rate of return required by the capital suppliers, based on the risk they perceive from our use, and based on the returns available elsewhere at equivalent risk.

\[ \$ \text{NPV}_0 = \Delta\$\text{PWE}_0 = -|\text{IVS}_0| + \sum_{t=0}^{T} (\text{kf*}) \text{ of all differential net after-tax operating cash flows.} \]

But cash flows are not tracked by accountants, so very few people comprehend NPV.

\[ \$ \text{EVA} = \text{income earned} - (\text{cost of capital} \times \text{investment}) \]

\[ \$ \text{EP} = (\text{actual ROI} - \text{cost of capital}) \times \text{Capital Invested} \]
Case 2: Entrepreneurial financial management decision—changing financing to improve performance. Although financing is purchased in an efficient market and often has no effect on the value of the firm, financing can improve or degrade performance by changes in the quantity of financing, and by changes in the proportions of debt and equity used. The Limited finds that it must alter its financing plans by raising more debt capital than it had planned to do to support its investments which are necessary to achieve the growth it seeks within the constraints of its operating efficiency. The Limited is not able to improve its operating efficiency as much as it wants to do so as to increase its success. It uses the mechanism of forecasting to discover (in the Kirznerian sense) how much new debt it needs to multiply by financial leverage the operating efficiency it is constrained to in the sense of the duPont equation.

The interaction of dividend/reinvestment policy (what portion of cash flow from operations to save and invest and what portion to pay out as dividends and share repurchases) and financing policy (what amount of new capital to borrow and what amount to sell new equity for); also see the title of Part Five on page 413: “Payout Policy and Capital Structure”.

The equity holders of the firm bear all of the business risk plus the financial risk created by any debt financing the firm has: Equity Risk = Business Risk + Financial Risk. The financial risk depends on the ratio of debt to total assets $\Theta = \frac{VD}{VF}$. They bear this additional financial risk even if the financial leverage is “favorable” and the firm’s assets return a higher rate than the interest rate on the debt. Because of the additional risk borne by the equity holders due to the debt of the firm, the cost of equity capital, the rate of return the shareholders require because of the risk they bear, rises when $\Theta$ rises.

We use the Capital Asset Pricing Model

$$\text{CAPM: } ke = R_f + \beta (E[R_M] - R_f) + \varphi$$

to evaluate the return required by the shareholders due to the Systematic Risk they bear, as measured by the equity $\beta$ of the firm. The $\beta$ is called the “volatility” of the stock, the ratio of the change in excess stock return $(R_j - R_f)$ when the market excess return $(R_M - R_f)$ changes: $\beta_j = (R_j - R_f) / (R_M - R_f)$. The beta is measured by linear regression of the excess return of the stock against the excess return of the market. (If the shareholders do not hold the firm’s stock as part of a perfectly-well-diversified portfolio, then they bear additional risk, “unique non-systematic risk” which adds an additional risk premium $\varphi$ added to the systematic risk, so the cost of equity capital will be still higher.)

Because the additional debt added to the capital structure imposes greater risk on the stockholders causing them to increase their cost of equity capital, the additional debt and the addition to $\Theta$ must increase the $\beta$ of the firm in order to achieve a higher value of $ke$ from the Capital Asset Pricing Model: $\beta = a$ function of $\Theta$. Higher values of $\Theta$ reflecting more debt financing raise the value of $\beta$ of the equity of the firm, thereby raising the cost of equity capital. There may also be an increase in the non-systematic unique risk borne by non-perfectly-diversified equity holders of the stock due to the increase in $\Theta$, but we have no theory of measurement for the unique risk.

If the capital structure ($\Theta = \frac{VD}{VF}$, the portion of the firm financed by debt) is altered, the cost of capital $[k^*f = \Theta k^* + (1-\Theta) ke]$ will change, and this change in the cost of capital will change the
Net Present Value of each newly-accepted project, and may also alter the identity of projects accepted because of the change in NPV across the zero border. The change in the capital structure alters the apportionment of risk between the debt holders and the equity holders, causing their respective costs of capital to change, and these changes in the individual costs of capital cause the overall weighted-average cost of capital to change, both by changes in individual ke and kd, but also by a change in the weighting factor \( \phi \). The risk apportionment changes because debt and equity have different orders of payment of owed cash: the debt is always paid first, before any payment can be made to equity. Hence, debt is always less risky than equity. Increases in the capital structure ratio, caused by a relative increase in the portion of financing due to debt, tend therefore to reduce the weighted-average cost of capital. Also, because of the tax savings on cash paid out as interest, compared with the cash paid out as dividends, the net-after-tax cost of debt capital to the firm is always smaller than the net cost of equity capital to the firm. However, as the portion of debt in the capital structure rises, increasing the risk borne by the equity holders, the equity holders’ required rate increases very rapidly, so that even though the portion of equity decreases, the overall cost of equity rises greatly, and after a time, the overall weighted-average cost of capital will rise. At very high debt ratios, \( kf^* > ku \).

But the attraction of debt is not only its relatively low cost, but also the magnification it accomplishes in the return to equity when financial leverage is “favorable”; i.e., when the interest rate on the debt is smaller than the pre-tax return on assets (\( kd < EBIT/TA \)). When financial leverage is favorable, debt in the capital structure raises ROE (\( = NI/NW \)) above the return on assets ROA (\( = NI/TA \)). (ROA is also called the “return on investment” ROI.) Remember the DuPont equation:

\[
ROE = \frac{NI}{NW} = \left( \frac{S}{TA} \right) \times \left( \frac{NI}{S} \right) \times \left( \frac{TA}{NW} \right) = \left( \frac{NI}{TA} \right) \times \left( \frac{TA}{NW} \right) = \frac{ROA}{1-\phi}
\]

Case 2 forecasts income statement, balance sheet, and cash flow statement for The Gap and Limited Brands for years 13-17 using forecast assumptions given on page 796. These forecasts are unsettling to the managers of Limited Brands because the ROA = NIAT/TA of The Gap exceeds that of Limited Brands because of The Gap’s larger profit margin on sales (The Gap has better control of operating expenses than does Limited Brands, especially a lower ratio of COGS/Sales for The Gap than for Limited Brands—this means that The Gap purchases goods at lower prices than can Limited Brands, or sells goods at higher prices than does Limited Brands, so The Gap’s profit margin on sales (Sales – COGS) / Sales or (Sales – COGS) / COGS is larger than is that of Limited Brands). This is an operating superiority on the part of The Gap. The Gap then exploits this operating superiority further by having a larger capital structure ratio (Debt/Total Assets) than does Limited Brands, and this larger financial leverage factor magnifies the Gap’s ROA into a larger ROE for The Gap than the smaller financial leverage factor of Limited Brands can magnify Limited’s smaller ROA into a still-smaller ROE. Although The Gap has a higher degree of financial leverage than does Limited Brands, Gap’s financial risk does not seem to be significantly larger than that of Limited, and because the debt of The Gap is of lower interest rate than the Return on Assets of The Gap, The Gap’s Financial Leverage is “favorable”, causing the enhancement of The Gap’s Return on Equity. It seems likely also that the degree of favorability of The Gap’s financial leverage (Gap ROA – Gap kd) exceeds the degree of favorability of Limited Brands’s financial leverage, causing a greater magnification of returns to Gap’s equity than the magnification achieved by Limited. So The Gap has two beneficial effects of its financial leverage: 1) a higher debt ratio than Limited; and 2) more efficient favorability of financial leverage. The overall risk of The Gap is probably no larger than that of Limited Brands, even though the capital structure ratio of The Gap is higher, because the operating leverage of The Gap is smaller and the business risk of The Gap is smaller than that of Limited Brands. So The Gap has a financial superiority over Limited Brands in addition to its operational superiority. The higher degree of financial leverage of The Gap causes its interest coverage ratio to be smaller than that of Limited Brands, but the additional risk of debt
apparently causes The Gap no distress. The Gap has a higher ratio of cash flow from operations to current liabilities than does Limited, as a result of Gap’s superior operations, control of operating expenses, and pricing margin. All of this causes The Gap’s Return on Equity to be larger than the Return on Equity (NI/NW) of Limited Brands, and that higher ROE of The Gap will persist into the future. Limited Brands must serve its shareholders better by raising its Return on Equity.

The best course of action for Limited Brands to catch up to The Gap would be to reduce its purchase prices of goods and increase its selling prices, perhaps through enhanced marketing activity improving its brand image, or else targeting a higher-income market to sell to than it has, and perhaps raising the quality of its goods to enhance their selling prices. It might invest in new stores in better locations so as to enhance the markup on the goods sold in the new stores compared with those of the old stores. If it financed these new capital acquisitions with debt, then the debt ratio would be raised and that would increase the ROA—ROE magnification, so long as the new debt was used favorably. There would then be both an operating improvement and a financial improvement, both of which would tend to increase ROE.

But if operating improvements cannot be made, then financial improvements alone must be considered. The goal is to raise Limited Brands’ ROCE = NIAT/NW. In the absence of any improvements of operations, Limited Brands must either increase its debt or decrease its equity, both of which will cause an increase in the magnification factor between ROA and ROE. Three financial changes are suggested:

**Scenario 1:** stretching out the payment of accounts payable. This will tend to raise the capital structure as the amount of debt (although not interest-bearing) will increase, and the interest charges will not increase. No increase in equity will occur, so this appears to be only an increase in the capital-structure ratio, and one might expect the ROE to be raised by the larger financial leverage magnification factor. The interest-coverage ratio will not increase, so the financial risk will not increase. There may therefore be an increase in the financial magnification between ROA and ROE, which may tend to raise ROE, without any increase in interest expense and without any increase in financial risk. This action will reduce the need for as much cash as formerly was needed, but the only effect on operating expenses will be to raise them, as some cash-payment discounts are lost. COGS/Sales will rise to 0.635. This will reduce operating income before the ROA is acted on by the higher capital structure magnification factor; our pro-forma forecast shows that there will be a net decline in the ROE, so this strategy is not helpful. So our forecast shows that we should not pursue this strategy. The decrease in the profit margin from the increase in COGS more than offsets the favorable effect of the increased financial leverage.

**Scenario 2:** increase the amount of Long-term debt by increasing its growth rate. This will increase the capital structure leverage ratio and will increase the ROA—ROE magnification factor, tending to cause an increase in ROE. But the increase in interest expense will decrease the interest-coverage ratio, take a larger portion of operating cash flow away from the equity holders, and therefore increase the financial risk from both of those factors. The increase in interest expense will also reduce the ROA due to the increase in expense, tending to offset the increase in the financial magnification factor. If we followed on and used the new debt to purchase new more-profitable fixed assets, or if we used the new debt to buy goods with a higher re-sale value and a higher markup, we could have an increase in operating efficiency as well; but the “Scenario” only suggests a financial change of capital structure, so there are no associated real effects. Again, our forecast shows that we should not pursue this strategy: that Scenario 3 will not improve Limited’s ROCE.

**Scenario 3:** increase the capital structure ratio $\Theta$ by reducing the amount of outstanding equity by repurchasing outstanding shares of stock from the market. We seek to duplicate The Gap’s ratio of Debt-to-Equity by reducing the amount of equity outstanding. This will increase the financial leverage magnification factor between ROA and ROE and increase ROE further, with no reduction in the operating cash flow ROA because there will be no increase in operating expenses and no increase in interest expenses, as there will be no additional borrowing. The risk ratios do not change because the
amount of debt does not change. The risk borne by the stockholders will increase due to the increase in capital structure, but the only manifestation of that will be in the value of ke, which is not observable. This, then, should be a pure financial leverage effect which should increase Limited’s ROCE. But cash is needed to buy back the stock. Without selling new debt or borrowing short-term, there is not enough cash on hand to buy back that much stock, so the balance in the Cash account turns negative in the forecasted pro-forma balance sheet. This shows us that this proposed Scenario, although it might achieve a high ROCE, is imaginary, since it cannot be accomplished without borrowing the cash to complete it. So the forecast also shows us the problems of this “pure financial” strategy. Of course, if we borrow the money to buy back the stock, we increase the amount of debt, increase the interest expenses, and increase the risk. The increase in interest expense causes ROA to be reduced, tending to offset the effects of the magnification due to financial leverage. So we must exercise a more complicated forecast Scenario to determine whether it would be beneficial to borrow the money to buy back the stock.

This Case, Problem 10.7, is an exercise in entrepreneurial financial management: selecting the best future scenario to pursue, on the basis of the ROCE which can be achieved by Limited Brands, in competition with The Gap. The Baseline forecast is developed in Part (a) and reveals that you expect Limited Brands to perform significantly worse during the coming five years than you expect The Gap to do. Your beginning analysis of the Base situation is developed in Part (b): your problem is that Limited Brands will have a smaller profit rate and return on equity than The Gap will have. You seek to improve Limited Brands’ profit rate and rate of return on equity to match those of The Gap. Choose the best from the four Scenarios: Base (Scenario 0), Scenario 1, Scenario 2, and Scenario 3. Show using duPont analysis for each year 13-17 the causes of the ROCE of Limited Brands:

\[
ROE = \frac{NIAT}{NW} = \frac{ROA}{1 - \theta} = \frac{(NIAT/TA)}{(TA/NW)} = \frac{(NI/Sales)}{(Sales/TA)} \cdot \frac{(TA/NW)}{TA/NW}.
\]

The scenario is that you are the Chief Financial Officer of Limited Brands, and you have been asked by the President of the Company to recommend the proper course of action for Limited Brands to raise its return on equity and compete more effectively with The Gap. This exercise is in two parts: 10.7 Part a, and 10.7 Parts b, c, d. You and the CEO are dissatisfied with the Part (a) forecast because the return achieved is too low and the risk perceived is too high, and you believe Limited Brands cannot compete effectively against The Gap under these constraints. You wish to propose a better course of action. Write your cover letter and report to the Chief Executive Officer of Limited Brands, in your capacity as the Chief Financial Officer, recommend a course of action for Limited Brands to follow, and explain why.

Part (a), the first part of the case, is an exercise in:

1) preparation of pro-forma financial statements from the baseline assumptions; and analysis of the future success of the firm using ratios, focusing on profitability and risk. Profitability and risk ratios are defined inside the back cover of the book, Chapter 4 is on "Profitability Analysis," and Chapter 5 is on "Risk Analysis." Recall that Profitability Analysis is essentially duPont analysis of ROE, and risk has two dimensions: business risk, measured by fluctuations in EBIT; and financial risk, measured by the fluctuations in returns on equity or earnings per share.

\[
ROE = \frac{NIAT}{NW} = \frac{ROA}{1 - \theta} = \frac{(NIAT/TA)}{(TA/NW)} = \frac{(NI/Sales)}{(Sales/TA)} \cdot \frac{(TA/NW)}{TA/NW}.
\]

Parts (b), (c), and (d), are exercises in entrepreneurial financial management: using the pro-forma statements resulting from a set of specific assumptions regarding future actions to evaluate the plan for future action implicit in those assumptions, and thereby select a better plan than the one you had already conceived; then doing that again and again. When you finish, you have a much better plan than you had before because you have forecasted the future much more accurately than you had before and your specific actions have a better result than the previously-imagined specific actions could have had.
You know the set of actions will be successful because the \textit{pro-forma} statements show what the results will be, if the forecast assumptions are correct. Of course, making correct forecast assumptions is really difficult.

The specific parts of this process in (b), (c), and (d) are the following:

2) evaluating the forecasted results and the comparative success against competitors;

3) improving the operations of Limited Brands by choosing one of the four Scenarios as the one to implement to compete better with The Gap.

4) Analyzing the entrepreneurial success of the chosen course of action for Limited Brands and discussing the reasons for the claimed entrepreneurial success in the context of the comparison with The Gap, the major competitor of Limited Brands.

\textbf{Problem 10.7 Part a.}

Your assignment for Case 3—Part "a"—is first to prepare the two sets of pro-forma statements, one set for each firm, using the assumptions given on page 796, and using the pro-forma statements to evaluate the risk and profitability using the ratios defined inside the back cover of the book and listed on pages 260-261. Your main discussion concerns the various assumptions given and your evaluation of the quality and appropriateness of the assumptions as measured by the past history of each firm, the reasonableness of the assumptions for forecasting the firms' statements in the market and competitive environment which exists, and your conclusions regarding the risk and profitability as shown by the ratios you computed. Pick a few interesting things about the risk and profitability of each firm's pro-forma statements to summarize in the Executive Summary.

Your cover letter for the case report has two pages: the first page covers part (a); the second page covers parts (b), (c), (d). The answer to part (a) is that you have indeed forecasted the pro-forma income statements, balance sheets, and cash flow statements of each firm for the next five years, and evaluated the profitability and risk of each firm, according to the set of assumptions he gave you (in Exhibit 10.11). Now you are poised to be able to vary the assumptions and evaluate the best course of action for Limited Brands. In the cover letter, write one sentence for each firm describing the profitability and risk of the firm, a sentence comparing the profitability and risk of the two firms, and then list the annual sales, operating income, and net income after taxes of each firm for each year, also list the total assets and capital structure ratio \( \theta = \frac{VD}{VF} \) for each firm for each year, and list the owners' equity amounts for each firm for each year. This should make two tables, each with six rows and five columns of data. Also show for each firm for each year the values of \( \frac{NI}{S}, \text{ROA}, \text{ROCE}, \frac{CA}{CL}, \frac{QA}{CL}, \text{COGS/Inv} \).
We will do sections (b), (c), and (d) next; in the second page of your cover letter, evaluate the four Scenarios for Limited Brands as they produce return and risk compared with that of The Gap. In your Appendices and Report work out the alternative pro-forma statements and ratios of The Gap and Limited Brands for each of the three scenarios 1, 2, and 3. In working parts (b), (c), and (d), you will analyze the outcomes of all four (4) Scenarios; i.e., 0, 1, 2, and 3, for the effects on the profitability and risk of Limited Brands; and you will comment on whether any of the three new scenarios improves the risk and return situation for Limited Brands compared with the baseline Scenario 0 and compared with its competitor The Gap. Note that part (c) asks you to evaluate the impact of the three Scenarios on the ratios of only Year 14. The Profitability Ratios are defined inside the back cover of the book, on the left-hand page. In addition to evaluating the impact on Year 14 as the text requests, I want you also to evaluate the impact of each Scenario on the ratios for all the forecast periods; i.e., for Year 13, Year 14, Year 15, Year 16, and Year 17 and comment about your findings comparing the performance of Limited Brands with that of The Gap. Especially comment regarding whether any of the Scenarios 1, 2, or 3 improves on the risk and return of Limited Brands compared with The Gap as forecasted in the baseline Scenario 0. Make a recommendation regarding which set of assumptions should be implemented and why in Cover Page 2 and Summary.

Problem 10.7 b, c, d.

You are the Chief Financial Officer of Limited Brands, with ultimate authority for financial management of the firm, and you are seeking to achieve in Years 13-17 a higher Return on Common Shareholders' Equity (ROCE) than you have had in the past, and to equal or surpass the ROCE of your major competitor The Gap. It is now the end of Year 12, and you have completed in Part (a) your forecasts of the pro-forma financial statements and ratios of your firm Limited Brands and those of your prime competitor The Gap for Years 13-17. This existing forecast of Years 13-17 of Limited Brands which you developed in Part (a) is called the "Baseline Scenario of future events." You and the CEO are dissatisfied with this forecast because the return achieved is too low and the risk perceived is too high, and you believe Limited Brands cannot compete effectively against The Gap under these constraints. You wish to propose a better course of action. To achieve a higher ROCE then your present Baseline plans will, you must alter your own firm's future plans for operations and financing (as presently stated in the Baseline Scenario) so that the future pro-forma financial statements will create future values of the ratio ROCE which you find more acceptable than the future values of ROCE which are created by your present Baseline plans and forecasts developed in Part (a), and thereby increase the wealth of your equity holders.

Now you are poised to be able to vary the assumptions and evaluate the best course of action for Limited Brands.

You will consider a change in the future plans from the current Baseline Scenario or Forecast so that a new set of forecasts of future financial statements (a "new scenario") will be made for Years 13-17. If you judge the new set of forecasts to be superior to the existing baseline set of forecasts, then you will implement changes in the actions of the firm compared with the actions which were implicit in the existing set of forecasts, and which were indeed the plans only a few minutes ago. This implementation of new changes based on alternative forecasts, is the action of entrepreneurship.

Actually, in this case you will prepare three (3) new sets of forecasts and assumptions—Scenario 1, Scenario 2, and Scenario 3 all as defined in Part (c)—and you will choose the best of the four scenarios for Limited Brands: that is Part (d). Your selection of the best of the four scenarios will be based on the Return on Common Shareholders' Equity (ROCE) ratio; also consider the other four ratios specified in Part (c) and discuss them as well as the values of $\theta = VD/VF$ for each firm. In normal
situations, we would compute the value of equity provided by each alternative Scenario instead of merely looking at the return and risk ratios; however, that would require longer forecasts and computation of the cost of capital, and that would take much longer.

In sections (b), (c), and (d), we will work out the alternative pro-forma statements and ratios of The Gap and Limited Brands for each of the three scenarios 1, 2, and 3 and compare them with Scenario 0. In working parts (b), (c), and (d), you will analyze the outcomes of all four (4) Scenarios; i.e., 0, 1, 2, and 3, for the effects on the profitability and risk of Limited Brands; and you will comment on whether any of the three new scenarios improves the risk and return situation for Limited Brands compared with the baseline Scenario 0 and compared with its competitor The Gap.

To achieve a higher ROCE then your present Baseline plans will, you must alter your own firm's future plans for operations and financing (as presently stated in the Baseline Scenario) so that the future pro-forma financial statements will create future values of the ratio ROCE which you find more acceptable than the future values of ROCE which are created by your present Baseline plans and forecasts developed in Part (a), and thereby increase the wealth of your equity holders.

In working parts (b), (c), and (d), you will analyze the outcomes of all four (4) Scenarios; i.e., 0, 1, 2, and 3, for the effects on the profitability and risk of Limited Brands; and you will comment on whether any of the three new scenarios improves the risk and return situation for Limited Brands compared with the baseline Scenario 0 and compared with its competitor The Gap.

In Parts (b), (c), and (d), (you are still the Chief Financial Officer of Limited Brands, with ultimate authority for financial management of the firm, and) you are seeking to achieve in Years 13-17 a higher Return on Common Shareholders' Equity (ROCE) than Limited Brands has had in the past, and to equal or surpass the ROCE of your major competitor The Gap. It is vital that your firm not fall behind The Gap competitively or in the return provided to equity holders. It is now the end of Year 12, and you have completed in Part (a) your forecasts of the pro-forma financial statements and ratios of your firm Limited Brands and those of your prime competitor The Gap for Years 13-17. This existing forecast of Years 13-17 of Limited Brands which you developed in Part (a) is called the "Baseline Scenario of future events." You and the CEO are dissatisfied with this forecast because the return achieved is too low and the risk perceived is too high, and you believe Limited Brands cannot compete effectively against The Gap under these constraints. You wish to propose a better course of action. To achieve a higher ROCE then your present Baseline plans will, you must alter your own firm's future plans for operations and financing (as presently stated in the Baseline Scenario) so that the future pro-forma financial statements will create future values of the ratio ROCE which you find more acceptable than the future values of ROCE which are created by your present Baseline plans and forecasts developed in Part (a), and thereby increase the wealth of your equity holders.

In Parts (b), (c), and (d) you will:
(1) seek to improve the operations of Limited Brands compared with those of The Gap and compared with those of the Baseline Scenario by designing three new Scenarios; (2) create alternative pro-forma statements from these new alternative scenarios and compare the results of the three alternatives with those of the baseline using ratios; (3) choose one of the four proposed Scenarios as the one to implement to compete better with The Gap; and then (4) analyze the entrepreneurial success of the chosen course of action for Limited Brands, and discuss the reasons for the claimed entrepreneurial success of the chosen Scenario in the context of the comparison with The Gap, the major competitor of Limited Brands. Your goal is to improve the opportunities Limited Brands offers to its customers—as revealed by the operating success—and the opportunities Limited Brands offers to its stockholders—as revealed by the ROCE or the stock price. You will not be very successful in this endeavor: success
would require much more time and better matching of the different assumptions and forecast parameters, but you should get from this a feel for how the process is done.

Note that part (c) asks you to evaluate the impact of the three Scenarios on the ratios of only Year 14; In addition to evaluating the impact on Year 14 as the text requests, I want you also to evaluate the impact of each Scenario on the ratios for all the forecast periods; i.e., for Year 13, Year 14, Year 15, Year 16, and Year 17 and comment about your findings comparing the performance of Limited Brands with that of The Gap. I want you to evaluate each of these ratios for all years 13, 14, 15, 16, and 17. The Profitability Ratios are defined inside the back cover of the Stickney book, on the left-hand page, I have enclosed them with your handout; comment about your findings comparing the performance of Limited Brands with that of The Gap. Especially comment regarding whether any of the Scenarios 1, 2, or 3 improves on the risk and return of Limited Brands compared with The Gap as forecasted in the baseline Scenario 0. Make a recommendation regarding which set of assumptions should be implemented and why in Cover Page 2 and Summary.

This process is what is meant by the term "entrepreneurial financial management": changing the future operating and financial plans of the firm (for years 13-17) from the current plans (for years 13-17) to achieve with the new plans improvement in the wealth of the equity suppliers compared with what the current plans can achieve. The new possible plans are forecasts of what you expect to occur, and you seek opportunities presently (i.e., prior to your creation of the new plans) unknown.

The new spectrum (wide-ranging alternative future forecasts) of Scenarios of plans must first be imagined or created; then each of the alternative new plans must be analyzed to see whether it is in fact better than the existing Baseline Scenario, and if so, which of the new Scenarios is the best. A new Scenario consists of newly-discovered opportunities which are unknown before the creation of the new Scenario by the entrepreneur. Whether the opportunities implicit in the Scenario are in fact superior to the presently-known opportunities must be determined by pro-forma analysis of the new Scenario and comparison of its future results with those that will result from the present plans.

This mode of thought has been most deeply analyzed by Prof. G. L. S. Shackle and Prof. Israel M. Kirzner, with strong contributions by Prof. Ludwig M. Lachmann. Ludwig Mises, of course, was the first teacher about entrepreneurship. David B. Hertz wrote seminally regarding the method of analysis comparing alternative new Scenarios. I shall give you Prof. Hertz's paper on simulation, and we shall simulate firm value in two cases this semester. Scenario analysis is just like simulation, only with many fewer variations—Scenario analysis usually has only a few alternative Scenarios; simulation has at least dozens of alternatives.

We can measure the improvement in the wealth of the equity suppliers from a particular Scenario by the fundamentally-analyzed stock price (estimated by discounting the future dividends and terminal value at the cost of equity capital) or more easily by the value of ROCE in the future pro-forma financial statements forecasted from each Scenario. As we do not have stock market information in this problem (i.e., future dividends and the cost of equity capital), we will not be able to estimate changes in the stock price, so we will focus on the ROCE; in real life, of course, we would forecast the future cash flows of the firm from each Scenario and discount the future Leveraged Cash Flows to Equity to estimate the effect of each Scenario on the stock price. This "change of the future plans" is with respect both to the present plans, as shown in the current financial statements, and to the existing (or "Baseline") set of future plans, which is shown by the current forecasts of future financial statements for Years 13-17 as developed in Part (a).

You will consider a change in the future plans from the current Baseline Scenario or Forecast so that a new set of forecasts of future financial statements (a "new scenario") will be made for Years 13-17. If you judge the new set of forecasts to be superior to the existing baseline set of forecasts, then you
will implement changes in the actions of the firm compared with the actions which were implicit in the existing set of forecasts, and which were indeed the plans only a few minutes ago.

This implementation of new changes based on alternative forecasts, is the action of entrepreneurship.

Actually, in this case you will prepare three (3) new sets of forecasts and assumptions—Scenario 1, Scenario 2, and Scenario 3 all as defined in Part (c)—and you will choose the best of the four scenarios for Limited Brands: that is Part (d). Your selection of the best of the four scenarios will be based on the Return on Common Shareholders' Equity (ROCE) ratio; also consider the other four ratios specified in Part (c) and discuss them as well as the values of $\theta = \frac{VD}{VF}$ for each firm.

On pages 260-261, the Rate of Return on Common Shareholders' Equity in Years 10, 11, and 12 (in reverse order) are shown for both Limited Brands and The Gap. Note that the ROCE of Limited Brands has declined from 19.0% to 13.7% over the three years, while the ROCE of The Gap collapsed from 59.2% in Year 10, through 34.0% in Year 11, to a tiny 0.8% in Year 12. The Gap has taken action to reverse that deadly trend, and your forecasts of the financial statements and ratios for Years 13-17 for The Gap, which you prepared in Part (a) of the problem, show that The Gap will achieve a ROCE of more than 28% in Years 13-17. The corresponding forecasts of Limited Brands for Years 13-17 which you prepared in Part (a) show only a modest increase in ROCE of Limited Brands in Year 13 to 15.9%, which will remain constant, while the ROCE of The Gap will be maintained at almost twice Limited Brands' height. This is a precarious competitive situation for Limited Brands to be in, first because it is expected to provide a smaller ROCE to its shareholders than will The Gap, leading to a fall in the stock price of Limited Brands, and second because this small ROCE shows inappropriate product pricing, markup, cost of goods, cost of operations, and capital structure for Limited Brands in its competition with The Gap: it is providing inferior opportunities to its customers compared with the opportunities provided by The Gap. You wish to improve the future performance of Limited Brands—to both your shareholders and to your customers—by creating a new set of more-successful plans than those of the Baseline Scenario of Part (a), and then implementing this newly-created set of plans to achieve the improvement of Limited Brands' performance.

Your beginning analysis of the Base situation is developed in Part (b): your problem is that Limited Brands will have a smaller profit rate and return on equity than The Gap will have.

This Case, Problem 10.7, is an exercise in selecting the best future scenario to pursue, on the basis of the ROCE which can be achieved by Limited Brands, in competition with The Gap. Choose the best from the four Scenarios: Base (Scenario 0), Scenario 1, Scenario 2, and Scenario 3. Show using duPont analysis for each year 13-17 the causes of the ROCE of Limited Brands:

$$\text{ROE} = \frac{\text{NIAT}}{\text{NW}} = \frac{\text{ROA}}{1 - \theta} = \frac{(\text{NIAT/T}\text{A})}{(\text{TA/N}\text{W})} = \frac{(\text{NI/Sales})}{(\text{Sales/TA})} \frac{(\text{TA/NW})}{.}$$

In Parts (b), (c), and (d) you will:

(1) seek to improve the operations of Limited Brands compared with those of The Gap and compared with those of the Baseline Scenario by designing three new Scenarios; (2) create alternative pro-forma statements from these new alternative scenarios and compare the results of the three alternatives with those of the baseline using ratios; (3) choose one of the four proposed Scenarios as the one to implement to compete better with The Gap; and then (4) analyze the entrepreneurial success of the chosen course of action for Limited Brands, and discuss the reasons for the claimed entrepreneurial success of the chosen Scenario in the context of the comparison with The Gap, the major competitor of Limited Brands. Your goal is to improve the opportunities Limited Brands offers to its customers—as revealed by the operating success—and the opportunities Limited Brands offers to its stockholders—as revealed by the ROCE or the stock price.

Write your cover letter to the Chief Executive Officer of Limited Brands, in your capacity as the Chief Financial Officer. Your answer to this part is that you have selected one Scenario (Either the Baseline, or No. 1, No. 2, or No. 3) to pursue, and give the identity of and reason for your selection in terms of the achievement of ROCE. In your cover letter, you may show a table for The Gap and the four Limited Brands Scenarios comparing the ROCE, the Profit Margin for ROCE, the Return on Assets...
ROA, the Capital Structure Ratio $\theta = TL/TA$, the Capital Structure Leverage Ratio, and the Cash Flow from Operations / Average Total Liabilities. You may also opine whether or not additional forecasted Scenarios might be even more helpful.

Note that part (c) asks you to evaluate the impact of the three Scenarios on the ratios of only Year 14; I want you instead to evaluate each of these ratios for all years 13, 14, 15, 16, and 17. The Profitability Ratios are defined inside the back cover of the book, on the left-hand page; comment about your findings comparing the performance of Limited Brands with that of The Gap. Especially comment regarding whether any of the Scenarios 1, 2, or 3 improves on the risk and return of Limited Brands compared with The Gap as forecasted in the baseline Scenario 0.

Make a recommendation regarding which set of assumptions should be implemented, and why.

The scenario is that you are the Chief Financial Officer of Limited Brands, and you have been asked by the President/CEO to recommend the proper course of action for Limited Brands to raise its return on equity and compete more effectively with The Gap. Write your cover letter to the Chief Executive Officer of Limited Brands, in your capacity as the Chief Financial Officer, recommend a course of action for Limited Brands to follow, and explain why.

This Case is an exercise in:

1) Part (a): preparation of pro-forma financial statements from the baseline assumptions; and analysis of the future success of the firm using ratios, focusing on profitability and risk. Profitability and risk ratios are defined inside the back cover of the book, Chapter 4 is on "Profitability Analysis," and Chapter 5 is on "Risk Analysis." Recall that Profitability Analysis is essentially duPont analysis of ROE, and risk has two dimensions: business risk, measured by fluctuations in EBIT; and financial risk, measured by the increase in fluctuations in returns on equity or earnings per share compared with the fluctuations in EBIT; financial risk is caused by $\Theta = VD/VF$.

Parts (b), (c), and (d), are exercises in entrepreneurial financial management: using the pro-forma statements resulting from a set of specific assumptions regarding future actions to evaluate the plan for future action implicit in those assumptions, and thereby select a better plan than the one you had already conceived; then doing that again and again. When you finish, you have a much better plan than you had before because you have forecasted the future more accurately than you had before and your specific actions have a better result than the previously-imagined specific actions could have had. You know the set of actions will be successful because the pro-forma statements show what the results will be, if the forecast assumptions are correct. Of course, making correct forecast assumptions is really difficult.

The specific parts of this process are the following:

2) creating alternative pro-forma statements from alternative scenarios and comparing the results of the three alternatives with those of the baseline by using ratios;

3) improving the operations of Limited Brands by choosing one of the four Scenarios as the one to implement to compete better with The Gap.

4) Analyzing the entrepreneurial success of the chosen course of action for Limited Brands and discussing the reasons for the claimed entrepreneurial success in the context of the comparison with The Gap, the major competitor of Limited Brands.

Your assignment for Case 2—Part "a"—is first to prepare the two sets of pro-forma statements, one set for each firm, using the assumptions given on page 796, and using the pro-forma statements to evaluate the risk and profitability using the ratios defined inside the back cover of the book and listed on pages 260-261. Your main discussion concerns the various assumptions given and your evaluation of the quality and appropriateness of the assumptions as measured by the past history of each firm, the reasonableness of the assumptions for forecasting the firms' statements in the market and competitive environment which exists, and your conclusions regarding the risk and profitability as shown by the ratios you computed. Pick a few interesting things about the risk and profitability of each firm's pro-forma statements to summarize in the Executive Summary.

Your cover letter has two pages: the first page covers part (a); the second page covers parts (b), (c), (d). The answer to part (a) is that you have indeed forecasted the pro-forma income statements, balance sheets, (and cash flow statements) of each firm for the next five years, and evaluated the profitability and risk of each firm, according to the set of assumptions he gave you (in Exhibit 10.11). Now you are poised to be able to vary the assumptions and evaluate the best course of action for Limited Brands. In the cover letter, write one sentence for each firm describing the profitability and risk of the firm, a sentence comparing the profitability and risk of the two firms, and then list the annual sales, operating income, and net income after taxes of each firm for each year, also list the total assets and capital structure ratio $\theta = VD/VF$ for each firm for each year, and list the owners' equity amounts for each firm for each year. This should make two tables, each with six rows and five columns of data. Also show for each firm for each year the values of NI/S, ROA, ROCE, CA/CL, QA/CL, COGS/Inv.

Follow the instructions for (a), and prepare pro-forma financial statements: income statements, balance sheets, (and cash flow statements) for The Gap and for Limited Brands, each for years 13, 14, 15, 16, and 17. Construct the pro-forma
You are the Chief Financial Officer of Limited Brands, with ultimate authority for financial management of the firm, and you are seeking to achieve in Years 13-17 a higher Return on Common Shareholders’ Equity (ROCE) than you have had in the past, and to equal or surpass the ROCE of your major competitor The Gap. It is now the end of Year 12, and you have completed in Part (a) your forecasts of the pro-forma financial statements and ratios of your firm Limited Brands and those of your prime competitor The Gap for Years 13-17. This existing forecast of Years 13-17 of Limited Brands which you developed in Part (a) is called the "Baseline Scenario of future events." You and the CEO are dissatisfied with this forecast because the return achieved is too low and the risk perceived is too high, and you believe Limited Brands cannot compete effectively against The Gap under these constraints. You wish to propose a better course of action. To achieve a higher ROCE then your present Baseline plans will, you must alter your own firm's future plans for operations and financing (as presently stated in the Baseline Scenario) so that the future pro-forma financial statements will create future values of the ratio ROCE which you find more acceptable than the future values of ROCE which are created by your present Baseline plans and forecasts developed in Part (a), and thereby increase the wealth of your equity holders.

In working parts (b), (c), and (d), you will analyze the outcomes of all four (4) Scenarios; i.e., 0, 1, 2, and 3, for the effects on the profitability and risk of Limited Brands; and you will comment on whether any of the three new scenarios improves the risk and return situation for Limited Brands compared with the baseline Scenario 0 and compared with its competitor The Gap. Note that part (c) asks you to evaluate the impact of the three Scenarios on the ratios of only Year 14. The Profitability Ratios are defined inside the back cover of the book, on the left-hand page. In addition to evaluating the impact on Year 14 as the text requests, I want you also to evaluate the impact of each Scenario on the ratios for all the forecast periods; i.e., for Year 13, Year 14, Year 15, Year 16, and Year 17 and comment about your findings comparing the performance of Limited Brands with that of The Gap. Especially comment regarding whether any of the Scenarios 1, 2, or 3 improves on the risk and return of Limited Brands compared with The Gap as forecasted in the baseline Scenario 0. Make a recommendation regarding which set of assumptions should be implemented and why in Cover Page 2 and Summary.

This process is what is meant by the term "entrepreneurial financial management": changing the future operating and financial plans of the firm (for years 13-17) from the current plans (for years 13-17) to achieve with the new plans improvement in the wealth of the equity suppliers compared with what the current plans can achieve. The new possible plans are forecasts of what you expect to occur, and you seek opportunities presently (i.e., prior to your creation of the new plans) unknown.

The new spectrum (wide-ranging alternative future forecasts) of Scenarios of plans must first be imagined or mentally created; then each of the alternative new plans must be analyzed to see whether it is in fact better than the existing Baseline Scenario, and if so, which of the new Scenarios is the best. A new Scenario consists of newly-discovered opportunities which are unknown before the creation of the new Scenario by the entrepreneur. Whether the opportunities implicit in the Scenario are in fact superior to the presently-known opportunities must be determined by pro-forma analysis of the new scenario and comparison of its future results with those that will result from the present plans.

This mode of thought has been most deeply analyzed by Prof. G. L. S. Shackle and Prof. Israel M. Kirzner, with strong contributions by Prof. Ludwig M. Lachmann. Ludwig Mises, of course, was the first teacher about entrepreneurship. David B. Hertz wrote seminally regarding the method of analysis comparing alternative new Scenarios using simulation analysis to reduce forecasting risk (the risk of being wrong in the forecasting of parameters affecting the future actions). I shall give you Prof. Hertz's paper on simulation, and we shall simulate firm value in the final case this semester, Rocky Mountain Advanced Genome. Scenario analysis is just like simulation, only with many fewer variations—Scenario analysis usually has only a few alternative Scenarios; simulation has at least dozens of alternatives.

We can measure the improvement in the wealth of the equity suppliers from a particular Scenario by the fundamentally-analyzed stock price (estimated by discounting the future dividends and terminal value at the cost of equity capital) or more easily by the value of ROCE in the future pro-forma financial statements forecasted from each Scenario. As we do not have stock market information in this problem (i.e., future dividends and the cost of equity capital), we will not be able to estimate changes in the stock price, so we will focus on the ROCE; in real life, of course, we would forecast the future cash flows of the firm from each Scenario and discount the future Leveraged Cash Flows to Equity to estimate the effect of each Scenario on the stock price. This "change of the future plans" is with respect both to the present plans, as shown in the current financial statements, and to the existing (or "Baseline") set of future plans, which is shown by the current forecasts of future financial statements for Years 13-17 as developed in Part (a).

You will consider a change in the future plans from the current Baseline Scenario or Forecast so that a new set of forecasts of future financial statements (a "new scenario") will be made for Years 13-17. If you judge the new set of forecasts to be superior to the existing baseline set of forecasts, then you will implement changes in the actions of the firm compared with the actions which were implicit in the existing set of forecasts, and which were indeed the plans only a few minutes ago.

This implementation of new changes based on alternative forecasts, is the action of entrepreneurship.
Actually, in this case you will prepare three (3) new sets of forecasts and assumptions—Scenario 1, Scenario 2, and Scenario 3 all as defined in Part (c)—and you will choose the best of the four scenarios for Limited Brands: that is Part (d). Your selection of the best of the four scenarios will be based on the Return on Common Shareholders’ Equity (ROCE) ratio; also consider the other four ratios specified in Part (c) and discuss them as well as the values of $\theta = \frac{VD}{VF}$ for each firm. In normal situations, we would compute the value of equity provided by each alternative Scenario instead of merely looking at the return and risk ratios; however, that would require longer forecasts and computation of the cost of capital, and that would take much longer.

On pages 260-261, the Rate of Return on Common Shareholders’ Equity in Years 10, 11, and 12 (in reverse order) are shown for both Limited Brands and The Gap. Note that the ROCE of Limited Brands has declined from 19.0% to 13.7% over the three years, while the ROCE of The Gap collapsed from 59.2% in Year 10, through 34.0% in Year 11, to a tiny 0.8% in Year 12. The Gap has taken action to reverse that deadly trend, and your forecasts of the financial statements and ratios for Years 13-17 for The Gap, which you prepared in Part (a) of the problem, show that The Gap will achieve a ROCE of more than 28% in Years 13-17. The corresponding forecasts of Limited Brands for Years 13-17 which you prepared in Part (a) show only a modest increase in ROCE of Limited Brands in Year 13 to 15.9%, which will remain constant, while the ROCE of The Gap will be maintained at almost twice Limited Brands' height. This is a precarious competitive situation for Limited Brands to be in, first because it is expected to provide a smaller ROCE to its shareholders than will The Gap, leading to a fall in the stock price of Limited Brands, and second because this small ROCE shows inappropriate product pricing, markup, cost of goods, cost of operations, and capital structure for Limited Brands in its competition with The Gap: it is providing inferior opportunities to its customers compared with the opportunities provided by The Gap. You wish to improve the future performance of Limited Brands—to both your shareholders and to your customers—by creating a new set of more-successful plans than those of the Baseline Scenario of Part (a), and then implementing this newly-created set of plans to achieve the improvement of Limited Brands’ performance.

Your beginning analysis of the Base situation is developed in Part (b): your problem is that Limited Brands will have a smaller profit rate and return on equity than The Gap will have.

This Case, Problem 10.7, is an exercise in selecting the best future scenario to pursue, on the basis of the ROCE which can be achieved by Limited Brands, in competition with The Gap. Choose the best from the four Scenarios: Base (Scenario 0), Scenario 1, Scenario 2, and Scenario 3. Show using duPont analysis for each year 13-17 the causes of the ROCE of Limited Brands:

$$ROE = \frac{NIAT}{NW} = \frac{NI}{AT} \times \frac{TA}{NW} = \frac{NI}{Sales} \times \frac{Sales}{TA} \times \frac{TA}{NW}.$$ 

In Parts (b), (c), and (d) you will:

(1) seek to improve the operations of Limited Brands compared with those of The Gap and compared with those of the Baseline Scenario by designing three new Scenarios; (2) create alternative pro-forma statements from these new alternative scenarios and compare the results of the three alternatives with those of the baseline using ratios; (3) choose one of the four proposed Scenarios as the one to implement to compete better with The Gap; and then (4) analyze the entrepreneurial success of the chosen course of action for Limited Brands, and discuss the reasons for the claimed entrepreneurial success of the chosen Scenario in the context of the comparison with The Gap, the major competitor of Limited Brands. Your goal is to improve the opportunities Limited Brands offers to its customers—as revealed by the operating success—and the opportunities Limited Brands offers to its stockholders—as revealed by the ROCE or the stock price. You will not be very successful in this endeavor: success would require much more time and better matching of the different assumptions and forecast parameters, but you should get from this a feel for how the process is done.

Write your cover letter to the Chief Executive Officer of Limited Brands, in your capacity as the Chief Financial Officer. Your answer to this part is that you have selected one Scenario (Either the Baseline, or No. 1, No. 2, or No. 3) to pursue, and give the identity of and reason for your selection in terms of the achievement of ROCE. In your cover letter, you may show a table for The Gap and the four Limited Brands Scenarios comparing the ROCE, the Profit Margin for ROCE, the Return on Assets ROA, the Capital Structure Ratio $\theta = \frac{TL}{TA}$, the Capital Structure Leverage Ratio, and the Cash Flow from Operations / Average Total Liabilities. You may also opine whether or not additional forecasted Scenarios might be even more helpful.

Note that part (c) asks you to evaluate the impact of the three Scenarios on the ratios of only Year 14; I want you instead to evaluate each of these ratios for all years 13, 14, 15, 16, and 17. The Profitability Ratios are defined inside the back cover of the book, on the left-hand page, I have enclosed them with your handout; comment about your findings comparing the performance of Limited Brands with that of The Gap. Especially comment whether any of the Scenarios 1, 2, or 3 improves on the risk and return of Limited Brands compared with The Gap as forecasted in the baseline Scenario 0.

Make a recommendation regarding which set of assumptions should be implemented, and why.
This Case, Problem 10.7, is an exercise in entrepreneurial financial management: selecting the best future scenario to pursue, on the basis of the ROCE which can be achieved by Limited Brands, in competition with The Gap. The Baseline forecast is developed in Part (a) and reveals that you expect Limited Brands to perform significantly worse during the coming five years than you expect The Gap to do. Your beginning analysis of the Base situation is developed in Part (b): your problem is that Limited Brands will have a smaller profit rate and return on equity than The Gap will have. You seek to improve Limited Brands’ profit rate and rate of return on equity to match those of The Gap. Choose the best from the four Scenarios: Base (Scenario 0), Scenario 1, Scenario 2, and Scenario 3. Show using duPont analysis for each year 13-17 the causes of the ROCE of Limited Brands:

\[ \text{ROE} = \frac{\text{NIAT}}{\text{NW}} = \frac{\text{ROA}}{(1 - \theta)} = \frac{\text{NIAT}}{\text{TA}} \times \frac{\text{TA}}{\text{NW}} = \frac{\text{NI}}{\text{Sales}} \times \frac{\text{Sales}}{\text{TA}} \times \frac{\text{TA}}{\text{NW}}. \]

The scenario is that you are the Chief Financial Officer of Limited Brands, and you have been asked by the President of the Company to recommend the proper course of action for Limited Brands to raise its return on equity and compete more effectively with The Gap. This exercise is in two parts: 10.7 Part a, and 10.7 Parts b, c, d. You and the CEO are dissatisfied with the Part (a) forecast because the return achieved is too low and the risk perceived is too high, and you believe Limited Brands cannot compete effectively against The Gap under these constraints. You wish to propose a better course of action. Write your cover letter and report to the Chief Executive Officer of Limited Brands, in your capacity as the Chief Financial Officer, recommend a course of action for Limited Brands to follow, and explain why.

Part (a), the first part of the case, is an exercise in:

1) preparation of pro-forma financial statements from the baseline assumptions; and analysis of the future success of the firm using ratios, focusing on profitability and risk. Profitability and risk ratios are defined inside the back cover of the book, Chapter 4 is on "Profitability Analysis," and Chapter 5 is on "Risk Analysis." Recall that Profitability Analysis is essentially duPont analysis of ROE, and risk has two dimensions: business risk, measured by fluctuations in EBIT; and financial risk, measured by the fluctuations in returns on equity or earnings per share.

\[ \text{ROE} = \frac{\text{NIAT}}{\text{NW}} = \frac{\text{ROA}}{(1 - \theta)} = \frac{\text{NIAT}}{\text{TA}} \times \frac{\text{TA}}{\text{NW}} = \frac{\text{NI}}{\text{Sales}} \times \frac{\text{Sales}}{\text{TA}} \times \frac{\text{TA}}{\text{NW}}. \]

Parts (b), (c), and (d), are exercises in entrepreneurial financial management: using the pro-forma statements resulting from a set of specific assumptions regarding future actions to evaluate the plan for future action implicit in those assumptions, and thereby select a better plan than the one you had already conceived; then doing that again and again. When you finish, you have a much better plan than you had before because you have forecasted the future much more accurately than you had before and your specific actions have a better result than the previously-imagined specific actions could have had. You know the set of actions will be successful because the pro-forma statements show what the results will be, if the forecast assumptions are correct. Of course, making correct forecast assumptions is really difficult.

The specific parts of this process in (b), (c), and (d) are the following:

2) creating alternative pro-forma statements from alternative scenarios and comparing the results of the three alternatives with those of the baseline by using ratios;

3) improving the operations of Limited Brands by choosing one of the four Scenarios as the one to implement to compete better with The Gap.

4) Analyzing the entrepreneurial success of the chosen course of action for Limited Brands and discussing the reasons for the claimed entrepreneurial success in the context of the comparison with The Gap, the major competitor of Limited Brands.
History of The Gap and Limited Brands

On pages 260-261, the Rate of Return on Common Shareholders' Equity in Years 10, 11, and 12 (in reverse order) are shown for both Limited Brands and The Gap. Note that the ROCE of Limited Brands has declined from 19.0% to 13.7% over the three years, while the ROCE of The Gap collapsed from 59.2% in Year 10, through 34.0% in Year 11, to a tiny 0.8% in Year 12. The Gap has taken action to reverse that deadly trend, and your forecasts of the financial statements and ratios for Years 13-17 for The Gap, which you prepared in Part (a) of the problem, show that The Gap will achieve a ROCE of more than 28% in Years 13-17. The corresponding forecasts of Limited Brands for Years 13-17 which you prepared in Part (a) show only a modest increase in ROCE of Limited Brands in Year 13 to 15.9%, which will remain constant, while the ROCE of The Gap will be maintained at almost twice Limited Brands' height. This is a precarious competitive situation for Limited Brands to be in, first because it is expected to provide a smaller ROCE to its shareholders than will The Gap, leading to a fall in the stock price of Limited Brands, and second because this small ROCE shows inappropriate product pricing, markup, cost of goods, cost of operations, and capital structure for Limited Brands in its competition with The Gap: it is providing inferior opportunities to its customers compared with the opportunities provided by The Gap. You wish to improve the future performance of Limited Brands—to both your shareholders and to your customers—by creating a new set of more-successful plans than those of the Baseline Scenario of Part (a), and then implementing this newly-created set of plans to achieve the improvement of Limited Brands' performance.

The Cover Letter

Write your cover letter to the President and Chief Executive Officer of Limited Brands, in your capacity as the Chief Financial Officer. Your cover letter has two pages: the first page covers part (a); the second page covers parts (b), (c), (d).

Your answer is that you have:
1. indeed forecasted the pro-forma income statements, balance sheets, and cash flow statements of each firm for the next five years, and evaluated the profitability and risk of each firm, according to the set of assumptions he gave you (in Exibit 10.11). In the cover letter, write one sentence for each firm describing the profitability and risk of the firm, a sentence comparing the profitability and risk of the two firms, and then list the annual sales, operating income, and net income after taxes of each firm for each year, also list the total assets and capital structure ratio \( q = \frac{VD}{VF} \) for each firm for each year. This should make two tables, each with six rows and five columns of data. Also show for each firm for each year the values of \( \frac{NI}{S}, \text{ROA}, \text{ROCE}, \frac{CA}{CL}, \frac{QA}{CL}, \text{COGS/Inv} \).
2. selected one Scenario (Either the Baseline, or No. 1, No. 2, or No. 3) to pursue, and give the identity of and reason for your selection in terms of the achievement of ROCE.

In the second page of your cover letter, evaluate the four Scenarios for Limited Brands as they produce return and risk compared with that of The Gap. Make a recommendation regarding which set of assumptions should be implemented and why in Cover Page 2 and Summary.

In your cover letter second page, you may show a table for The Gap and the four Limited Brands Scenarios comparing the ROCE, the Profit Margin for ROCE, the Return on Assets ROA, the Capital Structure Ratio \( \theta = \frac{TL}{TA} \), the Capital Structure Leverage Ratio, and the Cash Flow from Operations / Average Total Liabilities. You may also opine whether or not additional forecasted Scenarios might be even more helpful.

Analysis and Report:

In your Appendices and Report work out the alternative pro-forma statements and ratios of The Gap and Limited Brands for each of the three scenarios 1, 2, and 3. In working parts (b), (c), and (d), you will analyze the outcomes of all four (4) Scenarios; i.e., 0, 1, 2, and 3, for the effects on the
profitability and risk of Limited Brands; and you will comment on whether any of the three new scenarios improves the risk and return situation for Limited Brands compared with the baseline Scenario 0 and compared with its competitor The Gap. Note that part (c) asks you to evaluate the impact of the three Scenarios on the ratios of only Year 14. The Profitability Ratios are defined inside the back cover of the book, on the left-hand page. In addition to evaluating the impact on Year 14 as the text requests, I want you also to evaluate the impact of each Scenario on the ratios for all the forecast periods; i.e., for Year 13, Year 14, Year 15, Year 16, and Year 17 and comment about your findings comparing the performance of Limited Brands with that of The Gap. Especially comment regarding whether any of the Scenarios 1, 2, or 3 improves on the risk and return of Limited Brands compared with The Gap as forecasted in the baseline Scenario 0.

Entrepreneurial Financial Management

This process is what is meant by the term "entrepreneurial financial management": changing at this moment, the end of year 12, the future operating and financial plans of the firm (for years 13-17) from the current plans (for years 13-17), which have not yet been implemented, to achieve with the new plans improvement in the wealth of the equity suppliers compared with what the current-but-yet-un-implemented plans can achieve. The new possible plans are forecasts of what you expect to occur, and you seek opportunities presently (i.e., at this moment, prior to your creation of the new plans) unknown. They are presently unknown because you have not yet imagined them. Once you imagine several alternative possible future plans, you must then select one plan to implement, and then you must implement it.

The new spectrum (wide-ranging alternative future forecasts) of Scenarios of plans must first be imagined or mentally created; then each of the alternative new plans must be analyzed to see whether it is in fact better than the existing Baseline Scenario, and if so, which of the new Scenarios is the best. A new Scenario consists of newly-discovered opportunities (mentally imagined by the entrepreneur) which are (i.e., had been) unknown (to the entrepreneur and, of course, to all others as well) before the creation of the new Scenario by the entrepreneur. Whether the opportunities implicit in the newly-imagined Scenario are in fact superior to the presently-known opportunities must be determined by pro-forma analysis of the new Scenario and comparison of its expected future results with those that will result from the present plans.

This mode of thought has been most deeply analyzed by Prof. G. L. S. Shackle and Prof. Israel M. Kirzner, with strong contributions by Prof. Ludwig M. Lachmann. Prof. Kirzner was the chairman of my dissertation committee. I spoke often with Prof. Lachmann, who has written admiringly of Prof. Shackle’s analysis, which I have read. Ludwig Mises, of course, was the first teacher about entrepreneurship. David B. Hertz wrote seminaly regarding the method of analysis comparing alternative new Scenarios using simulation analysis to reduce forecasting risk (the risk of being wrong in the forecasting of parameters affecting the future actions). I shall give you Prof. Hertz's paper on simulation, and we shall simulate firm value in the final case this semester, Rocky Mountain Advanced Genome. Scenario analysis is just like simulation, only with many fewer variations—Scenario analysis usually has only a few alternative Scenarios; simulation has at least dozens of alternatives. Although all of this work had long been completed by 1980, only recently is it beginning to be used in mainstream Finance.

We can measure the improvement in the wealth of the equity suppliers from a particular Scenario by the fundamentally-analyzed stock price (estimated by discounting the future dividends and terminal value at the cost of equity capital) or more easily by the value of ROCE in the future pro-forma financial statements forecasted from each Scenario. As we do not have stock market information in this problem (i.e., future dividends and the cost of equity capital), we will not be able to estimate changes in the stock
price, so we will focus on the ROCE; in real life, of course, we would forecast the future cash flows of the firm from each Scenario and discount the future Leveraged Cash Flows to Equity to estimate the effect of each Scenario on the stock price. This "change of the future plans" is with respect both to the present plans, as shown in the current financial statements, and to the existing (or "Baseline") set of future plans, which is shown by the current forecasts of future financial statements for Years 13-17 as developed in Part (a).

FINANCING IMPLICATIONS DEDUCED FROM BALANCE SHEET FORECASTS

When forecasting the Balance Sheet, usually the left-hand-side is forecasted first, so that the Total Assets required for the forecasted future course of action are first determined; then the Total of Liabilities and Equity is equal to Total Assets, and the analyst moves up the right-hand-side of the Balance Sheet, leaving Notes Payable (short-term debt) as the Plug. **This method has the effect of deducing the amount of new capital required to finance and support the assets required to achieve the forecast.** (Recall from Sustainable Growth theory that to increase sales, either the operating efficiency of the assets must be increased—a very difficult task—or the quantity of assets must be increased; increasing the quantity of assets requires paying for the new assets; *i.e.*, financing them.) Making Notes Payable the Plug on the right-hand-side of the Balance Sheet deduces the quantity of new debt capital required to finance the implied new assets required to achieve the forecast, subject to the implicit assumptions regarding the operating efficiency of the assets.

If you deduce that Notes Payable is a positive algebraic amount on the right-hand-side of the Balance Sheet, that means that your firm must borrow new debt capital to finance the proposed actions and expected events by borrowing and using the new funds to purchase new capital assets and working capital, or to pay the development expenses. This is the general method used in Balance Sheet forecasting to determine the viability of a proposed future course of action: if the amount of new debt required is too large, then the proposed course of action will likely not succeed. Before forecasting the Balance Sheet with Notes Payable as the plug, it cannot be said how much new capital will be required to achieve the forecast activities and purchase the required assets: this is why forecasting only the Income Statement is never enough to see the future clearly. Forecasting the Income Statement without the Balance Sheet may tacitly assume a sharp and profound increase in operating efficiency which is not justified, or it may tacitly assume the firm can borrow whatever amount of debt it needs to finance the new assets required to achieve the forecasts, when in reality, no lender will commit such a quantity of funds because of the increased financial risk it will bear.

You may find that some forecasted values of Notes Payable are negative; *i.e.*, are algebraic negative amounts on the right-hand-side of the Balance Sheet. This implies that the other liabilities and equity amounts are too large for the assets forecasted, and Notes Payable must be negative to cause the Balance Sheet to Balance with the smaller amount of Total Assets. You can improve the Balance Sheet by adding the absolute value of your negative Notes Payable amount to the
Cash account on the LHS and replacing the negative Notes Payable amount with a zero. Remember then to add the newly-created cash to the totals of Current Assets and Total Assets. This action will keep the Balance Sheet in Balance if you handle it correctly, but with a larger Balance Sheet total than before. Following this adjustment, you will then see that the forecasted course of action creates new cash from operations, suggesting that the forecasted course of actions will in fact be successful. In the view of the Sustainable Growth Model, the firm will be creating cash available for disgorgement and will be growing at an actual rate smaller than the Sustainable-Rate-Assuming-No-Increase-in-Operating-Efficiency. The increase in the Sustainable Growth Rate will come about because of the new course of action assumed to be implemented, with new assets, a new business plan, newly improved operating efficiency, etc.
The interaction of dividend/reinvestment policy (what portion of cash flow from operations to save and invest and what portion to pay out as dividends and share repurchases) and financing policy (what amount of new capital to borrow and what amount to sell new equity for).

The equity holders of the firm bear all of the business risk plus the financial risk created by any debt financing the firm has: Equity Risk = Business Risk + Financial Risk. The financial risk depends on the ratio of debt to total assets \( \Theta = \frac{VD}{VF} \). They bear this additional financial risk even if the financial leverage is “favorable” and the firm’s assets return a higher rate than the interest rate on the debt. Because of the additional risk borne by the equity holders due to the debt of the firm, the cost of equity capital, the rate of return the shareholders require because of the risk they bear, rises when \( \Theta \) rises.

We use the Capital Asset Pricing Model

\[
ke = R_f + \beta (E[R_M] - R_f) + \varphi
\]

CAPM: to evaluate the return required by the shareholders due to the Systematic Risk they bear, as measured by the equity \( \beta \) of the firm. The \( \beta \) is called the “volatility” of the stock, the ratio of the change in excess stock return \( (R_j - R_f) \) when the market excess return \( (R_M - R_f) \) changes: 

\[
\beta_j = \frac{(R_j - R_f)}{(R_M - R_f)}.
\]

The beta is measured by linear regression of the excess return of the stock against the excess return of the market. (If the shareholders do not hold the firm’s stock as part of a perfectly-well-diversified portfolio, then they bear additional risk, “unique non-systematic risk” which adds an additional risk premium \( \varphi \) added to the systematic risk, so the cost of equity capital will be still higher.)

Because the additional debt added to the capital structure imposes greater risk on the stockholders causing them to increase their cost of equity capital, the additional debt and the addition to \( \Theta \) must increase the \( \beta \) of the firm in order to achieve a higher value of \( ke \) from the Capital Asset Pricing Model: \( \beta = \text{a function of } \Theta \). Higher values of \( \Theta \) reflecting more debt financing raise the value of \( \beta \) of the equity of the firm, thereby raising the cost of equity capital. There may also be an increase in the non-systematic unique risk borne by non-perfectly-diversified equity holders of the stock due to the increase in \( \Theta \), but we have no theory of measurement for the unique risk.

If the capital structure \( (\Theta = \frac{VD}{VF}, \text{the portion of the firm financed by debt}) \) is altered, the cost of capital \( [k_f^* = \Theta kd^* + (1-\Theta) ke] \) will change, and this change in the cost of capital will change the Net Present Value of each newly-accepted project, and may also alter the identity of projects accepted because of the change in NPV across the zero border. The change in the capital structure alters the apportionment of risk between the debt holders and the equity holders, causing their respective costs of capital to change, and these changes in the individual costs of capital cause the overall weighted-average cost of capital to change, both by changes in individual \( ke \) and \( kd \), but also by a change in the weighting factor \( \Theta \). The risk apportionment changes because debt and equity have different orders of payment of owed cash: the debt is always paid first, before any payment can be made to equity. Hence, debt is always less risky than equity. Increases in the capital structure ratio, caused by a relative increase in the portion of financing due to debt, tend therefore to reduce the weighted-average cost of capital. Also,
because of the tax savings on cash paid out as interest, compared with the cash paid out as dividends, the net-after-tax cost of debt capital to the firm is always smaller than the net cost of equity capital to the firm. However, as the portion of debt in the capital structure rises, increasing the risk borne by the equity holders, the equity holders’ required rate increases very rapidly, so that even though the portion of equity decreases, the overall cost of equity rises greatly, and after a time, the overall weighted-average cost of capital will rise. At very high debt ratios, $k_f^* > k_u$.

But the attraction of debt is not only its relatively low cost, but also the magnification it accomplishes in the return to equity when financial leverage is “favorable”; i.e., when the interest rate on the debt is smaller than the pre-tax return on assets ($k_d < EBIT/TA$). When financial leverage is favorable, debt in the capital structure raises ROE ($= NI/NW$) above the return on assets ROA ($= NI/TA$). (ROA is also called the “return on investment” ROI.) Remember the DuPont equation:

$$\text{ROE} = \frac{NI}{NW} = \frac{(S/TA) \times (NI/S)}{(TA/NW)} = \frac{(NI/TA) \times (TA/NW)}{1-\epsilon}$$

**EFFECT OF CAPITAL STRUCTURE ON BETA AND ON $ke$**


$$\beta_{\text{observed}} = \beta_U \left[ 1 + (1-\tau) \frac{VD}{VE} \right]$$

The capital structure of the firm is thought to influence the numerical value of beta.

Hamada derived the expanded Capital Asset Pricing Model, which takes account of the tax-deductibility of interest on corporate debt and the capital structure of the firm$^2$:

$$ke_j = R_f + \left\{ \left( E[R_M] - R_f \right) / \sigma_M^2 \right\} \left[ \rho_{ju,M} \sigma_{ju} \sigma_M \right] \left[ 1 + VD/VE \right] \left( 1-\tau_c \right)$$

where $ke_j$ = the cost of equity capital of the $j^{th}$ security, the rate required by the equity holders; $\beta$ indicates “unleveraged”; i.e., with debt = $0$; $R_f$ = the risk-free rate in the market; $E[R_M] = the$ expected rate of return on the market portfolio during the relevant period of time; $\sigma_M = the$ standard deviation of the probability distribution of possible market returns; $\rho_{ju,M} = the$ correlation coefficient between returns for security $j$ in the absence of leverage and the market portfolio; $\sigma_{ju} = the$ standard deviation of the probability distribution of possible returns for security $j$ in the absence of leverage; $VD/VE = ratio$ of value of debt to value of equity in market-value terms; $\tau_c = corporate$ income-tax rate.

The above equation can be simplified by using the ordinary definition of $\beta = \rho_{jM} \sigma_j / \sigma_M$

---

\[ \text{ke}_j = R_f + (E[R_M] - R_f) \beta_{ju} \{ 1 + (VD/VE) (1 - \tau_c) \} \]

where \( \beta_{ju} \) is the beta measuring the responsiveness of the excess return for the security in the absence of leverage to the excess return of the market portfolio. This value of beta, of course, cannot be observed if the firm has any debt. We must calculate the \( \beta_{ju} \) from the observed value of \( \beta_j \) using the equation below:

\[ \beta_{ju} = \beta_j / \{ 1 + (VD/VE) (1 - \tau_c) \} \]

In \( \text{ke}_j \), the premium for business risk is:

\[ (E[R_M] - R_f) \beta_{ju} \]

In \( \text{ke}_j \), the premium for financial risk is:

\[ (E[R_M] - R_f) \beta_{ju} \{ (VD/VE) (1 - \tau_c) \} \]

The measured, or observed, value of beta for the stock, \( \beta_j \), embodies both risks and is:

\[ \beta_j = \beta_{ju} \{ 1 + (VD/VE) (1 - \tau_c) \} \]

CASE 3

The “risk” we perceive in a proposed course of action is our perception of our ability to forecast accurately the outcome of the proposed course of action. If we believe that our forecasting ability is perfect, then we believe we know the outcome for certain, and we perceive no risk. This is true even if other people are perplexed about what will occur. If we believe we know, then we bear no risk, and our behavior is indistinguishable from that of a person who really does know.

If we believe that our forecasting ability is very weak, then we believe we do not know what might happen, and we perceive great risk. This is true even if other people know for sure what will happen. This is true even if the outcome is absolutely certain, but we do not perceive that certainty.

“Risk” is a second-order subjective perception, a second-order creation in our own mind. Risk is our opinion about our ability to forecast accurately. Risk is not merely about our actual ability to forecast accurately; risk is our opinion about that forecasting ability. Even if we cannot forecast accurately, if we believe we can forecast perfectly, we bear no risk and behave as we do in a situation of certainty.

The managers of the firm choose courses of action based on the effect on value of equity of the course of action. That choosing of a course of action is called a “decision.” The computation of the value of equity takes account of the alternative rates of return available to the equity suppliers of the firm on a risk-adjusted basis through the discounting of future expected cash flows at the cost of equity capital. The cost of equity capital is adjusted to take account of the differential risk of the different considered future courses of action. The computation of the value of equity takes account of the productivity of each alternative course of action by forecasting the stream of future cash flows which that action will achieve. This is a forecasting activity. We forecast the stream of future cash flows we expect and discount them at the risk-adjusted cost of capital to compute the present value. We choose the course of action to pursue by selecting the course with the highest present value.

But what if we make a mistake in our forecasts? We have still the problem of “forecasting risk”: the knowledge that the future is unknowable, and all we have done in our forecast is imagine it in our own minds in advance, using whatever forecasting expertise and intuition we have. But our imagining must be somewhat in error; i.e., particular forecasted parameters we have used may turn out
to be different numbers than we have assumed, and this will cause the outcome to be different—so
different that perhaps this course of action, which we had thought would create the greatest value,
actually will not, and would be seen afterward as a mistake.

**Risk, perceived ignorance of the future.** must be taken into account both in the numerator and in
the denominator of the valuation equation. In the denominator, it is taken into account in the numerical
value of the risk-adjusted discount rate, the “cost of capital,” the rate of return available elsewhere to our
capital suppliers on similar investments of similar risk. Both business risk and financial risk are taken
into account in \( k_f^* \) and in \( k_e \); we must remember also that the size of the risk-adjusted discount rate
depends on the perceptions of risk of our investors, and so those discount rates can also change with
changes in our investors’ mental states and perceptions. The weighted-average cost of capital \( k_f^* \) also
depends on the capital structure ratio \( \Theta \), and that can change as time passes, as the firm changes its
capital structure, either deliberately or accidentally, as in response to higher- or lower-than-expected
cash retentions from operations (which accrue only to the equity suppliers and therefore change the
present value of equity).

In the numerator of the valuation equation, risk is taken into account in the numerical value of
the cash flows forecasted for each future period and in the numerical value of the terminal value at the
end of the explicit forecasting period. The risks within the cash flows are the variability possible in the
various forecasting parameters which go into determining the cash flow: the possible variation in
receipts, in expenses, in interest payments, in new financing needed, in the growth rates of various future
cash flows, in the portions of sales which each expense comprises, in the inventory turnover ratio, in the
average collection period of receivables, etc. Each of the possible changes in all of these internal
parameters can and will change the size of the cash flow of a particular year, and the effects will be
different in different years. We cannot know what will happen to each of the forecasted parameters: the
future is unknowable; but we can estimate the likely range within which each of the parameters will
vary: the future is not unimaginable.

There are three methods to try to deal with forecasting risk so as to make more accurate
decisions (i.e., decisions which more accurately depict the actual future events and conditions which will
occur; decisions which are less likely to be considered mistakes):

1) **sensitivity analysis**—looking at the effect on our present value of different numerical
sizes of one important forecast parameter;

2) **scenario analysis**—looking at the effect on our present value of a few different forecast
scenarios, different sets of alternative forecast parameters, such as “best case”, “most likely case”, and
“worst case”, in which each “case” is a complete system of forecast parameters with different values; we
then compute an expected present value by multiplying the result of each case analysis by its estimated
probability of occurrence and adding the products of probability times case value to compute the
expected value;

3) **simulation**—looking at the effect on our present value of changes in all forecast
parameters within their possible ranges; this takes account of unspecified interactions among the
forecast parameters. We compute a present value for each vector of forecast parameter amounts; change
each forecast parameter amount and make a new vector and compute the present value for that vector.
We repeat the process hundreds of times, taking account of all possible changes in each forecast
parameter amount within the range of possible amounts of forecast parameters, and taking account of the
shape of the distribution of amounts of forecast parameters within the range. We plot a frequency
distribution of present value and choose the mean value or the most likely value as the risk-adjusted
value of the course of action. This takes into account the likelihood that different combinations of the forecast parameters can produce a similar present value.

The value of a business is the value now (i.e., the importance now, measured in dollars) of all of the future cash flows which the firm will generate during its entire future lifetime for the owners. Most commonly, those estimated and forecasted future cash flows are measured using the Leveraged Free Cash Flow to Equity method. Those future cash flows must be forecasted. Determining their value now depends on the rate of return now available for alternative investments of comparable risk borne by the owners if they should invest in other firms than this one; this estimate of the alternative rate of return available in other investments determines the discount rate used to value the forecasted cash flows of this business. This discount rate used to value the entire stream of future LFCFE’s is the cost of equity capital, ke, usually computed from the Capital Asset Pricing Model with an adjustment for unique risk in the situation; but instead of using the CAPM, an equity risk premium can be added to the required rate on debt.

Valuing a business is a delicate balance of capturing the actual growth of sales and dividends which will occur in the next few years of the future without over-estimating the long-term growth which can be achieved in the long run (which will cause the Gordon-Model computed Terminal Value to be too large) and thereby over-valuing the present value of the business because of the disproportionate share of present value of the firm contributed by the present value of the Terminal Value. Because they neglect to add the Terminal Value at the very end of the years of explicit forecast, many people underestimate the true value of the firm. You must always include the Terminal Value, and you must always discount it to the present. But be aware that the present value of the Terminal Value usually contributes about 80% or more of the total present value of the firm. This is especially true when using the Gordon Model to determine the Terminal Value. Because the Gordon Model tends to overstate the present value of a growing stream of cash flows, an arbitrary reduction factor (like “0.9”) may be used as a multiplier with the Gordon Model before the Terminal Value is discounted.

During those periods of time when there is a deep term structure of interest rates, you must remember to use a non-constant discount rate for cash flows expected in different future years to reflect the current term structure of rates. Most commonly, longer-term rates are larger than shorter-term rates; but that is not always the case.

The next issue to watch out for is changes in the risk of the firm, both the systematic risk (which will change the value of the beta factor in the Capital Asset Pricing Model for the cost of equity capital) and the non-systematic, or unique risk (which adds a premium for the unique non-systematic risk borne by the equity holder if the firm is not part of a well-diversified portfolio held by the owner). This is especially true if the issue is the embarking on a new line of business, such as that proposed for the Massachusetts Stove Company. With the Rocky Mountain Advanced Genome Company, as time passes, there may be changes in the riskiness of future cash flows as the company successfully develops technologies or products which are not now known firmly, or as the company’s competitive position changes, or as the company changes its future capital structure.
DEFINITIONS AND EQUATIONS FOR VALUATION:

Financial Free Cash Flows, UFCFF, LFCFE Are Used to Value the Firm and to Value Equity.

\[ ke = \text{cost of equity capital} = \text{the risk-adjusted rate of return required by the shareholders} \]

\[ ke = kd + \text{risk adjustment} = \frac{d_i}{P_0} + g = R_t + \beta (E[R_M] - R_t) \]

\[ kf^* = \text{the weighted-average cost of capital} = \theta kd (1 - \tau) + (1 - \theta) ke \]

\[ \text{LFCFE}_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t - \text{IVS}_t - \Delta \text{NWC}_t - \text{PP}_t + \text{NEC}_t + \text{NDC}_t + \text{NPS}_t - \text{Pfd Divs}_t \]

\[ \text{UFCFF}_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t + \text{Int}_t (1 - \tau) - \text{IVS}_t - \Delta \text{NWC}_t + \text{NDC}_t + \text{NEC}_t + \text{NPS}_t - \text{PfdDivs}_t \]

\[ \Delta \text{NWC}_t = \text{NWC}_t - \text{NWC}_{t-1}, \text{and} \text{NWC}_t = \text{CA}_t - \text{CL}_t \]

The value of the firm \( VF_0 \) is the present value to the owners of all the future benefits they will receive from the firm for the remainder of its lifetime forward from the present day. If we include "bondholders" in "owners", then the value of the debt is included in this value. If we do not include bondholders, then the owners are only the equity holders, and the value considers only the equity, so it is called Value of Equity, \( VE_0 \).

The value of the firm \( VF_0 \) is the present value to the owners (debt holders and equity holders) of all the future benefits they will receive from the firm for the remainder of its lifetime forward from the present day. The value of equity \( VE_0 \) is the value of the firm minus the value of debt: \( VE_0 = VF_0 - VD_0 \). The value of equity \( VE_0 \) is also the present value to the equity holders of all the future net benefits (dividends minus new equity capital supplied) they will receive from the firm for the remainder of its lifetime forward from the present day.

\[ VF_0 = \sum_{t=1}^{T} \left[ \frac{\text{UFCFF}_t}{(1 + kf^*)^t} \right] + \frac{TV_T}{(1 + kf^*)^T} \]

\[ TV_T = \text{Terminal Value at time } T = \frac{\text{UFCFF}_{T+1}}{(1 + g_\infty)} \]

\[ VE_0 = VF_0 - VD_0 \]

We calculate the Value of Equity \( VE_0 \) by discounting the series of the "Leveraged Free Cash Flows to Equity at each time point" (LFCFE) from \( t=1 \) through \( t=T \) plus the Terminal Value at \( T \), all by the cost of equity capital \( ke \). In this Problem, \( t1 = t1 \), and \( T \) is \( t2 \).

\[ VE_0 = \sum_{t=1}^{T} \left[ \frac{\text{LFCFE}_t}{(1 + ke)^t} \right] + \frac{TV_T}{(1 + ke)^T} \]

\[ TV_T = \text{Terminal Value at time } T = \frac{\text{LFCFE}_{T+1}}{(ke - g_\infty)} \]

\[ \text{LFCFE}_{T+1} = \text{LFCFE}_T (1 + g_\infty) \]

Free Cash Flow (FCF) or "Free Cash Flow to Equity" (FCFE) or "Leveraged Free Cash Flow to Equity" (LFCFE) is the annual cash flow the firm can use to pay common dividends after the company has made all the investments in fixed assets and working capital necessary to sustain ongoing operations and without adversely affecting the planned growth of the firm; i.e., after purchasing plant and equipment needed to maintain operations and achieve growth, and after
purchasing the net working capital required to operate the new plant and equipment. This
definition of "Free Cash Flow" subtracts the required debt service \((PP + I)\) for the year on the
way to the computation of FCF. We call this "Leveraged Free Cash Flow to Equity Suppliers", 
LFCFE. (Stickney)

\[
VE_0 = \sum_{t=1}^{T} \left[ \frac{(LFCFE_t)}{1 + ke} \right] + TV_T / (1 + ke)^T
\]

**TERMINAL VALUE** at time \(T = TV_T;\)

\(TV_T = \text{Terminal Value at time } T = \text{the value at a particular time point } T \text{ of all of the following cash}
flows for the remaining lifetime of the firm; i.e., beginning at \(T+1\) and going on to \(\infty\). The value
of a future cash flow occurring at some future time point, valued at a particular time point \(T\), is
computed by discounting that cash flow from its time point of occurrence back to the particular
time point \(T\) at the proper discount rate or "cost of capital". All of the present values at time \(T\) of
all of the future cash flows are then added together to get \(TV_T.\)

\(TV_T = \text{Terminal Value at time } T = \frac{\text{(Relevant Cash Flow}_{T+1)} / (ke - g)}\)

**COST OF CAPITAL, THE DISCOUNT RATE FOR FUTURE CASH FLOWS:**

1. Weighted-average cost of capital, \(kf^* = \theta kd (1 - \tau) + (1 - \theta) ke\)
   Used to discount UFCFF's.

2. Cost of Equity Capital,
   \(ke = Rf + \beta (E[RM] - Rf) + \text{non-systematic risk factors}\)
   Used to discount LFCFE's.
Case 4: Rocky Mountain Advanced Genome, Inc.

Report Solution Due: Tuesday July 1, 2008

Cover Letter: Present the alternative values of $\text{VE}_{12/31/95}$ of RMAG, and the value of 90% VE:

1) based on the RMAG forecasts, with TV in 2005, specified $g_c$, and $k_e = 0.20$;
2) based on the Big Sur forecasts, with TV in 2006, specified $g_c$, and $k_e = 0.20$;
3) based on your extension of one of these forecasts to a further-in-the-future year--
   TV in year specified, specified $g_c$, and $k_e = 0.20$; and
4) the expected value of your simulation of value based on your extended valuation (3) above
   using the same terminal year, and all of the variables specified with distributions of
   forecast parameters specified.

Executive Summary: Briefly discuss the assumptions and contents of the four valuation models; report the valuations.

Analysis: Discuss the four valuation models, discuss forecasting risk, discuss simulation of value, discuss your choice of the horizon date, discuss your choice of the preferred $g_c$ values you used, discuss Exhibit 6, and answer the remaining "task" questions, as I specify.

Appendix I, II, III, IV: Spreadsheets for each valuation model computing $\text{VE}_{12/31/95}$.

Valuation is the central concept of Finance. Finance is always forward-looking. Finance neglects historical data: "sunk costs are sunk" and sunk costs cannot affect value. Valuation is a division of Finance as well as the central concept. Valuation is always forward-looking. "Value" is the significance now of all the future benefits to be provided to the owners over the lifetime of the asset. Value has nothing whatever to do with historical events or conditions. Valuation is not a division of accounting, which is only concerned with historical data.

This is a case regarding Valuing the Enterprise. Valuing Rocky Mountain Advanced Genome; that is, discovering the market price for the entire firm, is your main task for this project. Your report will consist of text and spreadsheets. The text will state the value of the firm, discuss your assumptions in each of the valuations and choose among them, explain your method of analysis, and answer the questions posed. Write the report to Kate McGraw as a complete solution to the problem of what amount of cash Big Sur Capital Management Company should pay for a 90% equity interest in Rocky Mountain Advanced Genome on January 31, 1996. Rocky Mountain Advanced Genome will have no debt, so its capital structure ratio $\theta = 0.00$ and $k_f^* = k_e$. The cash flows are all Leveraged Free Cash Flows to Equity, $\text{LFCFE}_t$'s.
Your report should begin with a one-page formal cover letter to Kate McGraw giving your conclusion as to the value of the firm and a 90% portion of it based on your four valuations, with emphasis given to your simulated value conclusion; then you should have a two-page summary of the important issues affecting valuation; and then a full text report explaining in detail your valuation methodology and your solution to each problem and question posed; last an Appendix in which you provide all of the spreadsheets you created to value the firm, as specified below. Each spreadsheet should be a complete valuation, with a title, parameters, annual cash flows, discounted cash flows, and present value of the firm. You may put your assumptions and parametric values on the spreadsheet.

You will prepare three single-point valuations and one simulation:

1) \( VE_{12/31/1995} \) and 90% of \( VE_{12/31/1995} \) based on the RMAG forecasts;
2) \( VE_{12/31/1995} \) and 90% of \( VE_{12/31/1995} \) based on the Big Sur forecasts;
3) \( VE_{12/31/1995} \) and 90% of \( VE_{12/31/1995} \) based on your extension of one of these forecasts to a further-in-the-future year; and
4) \( VE_{12/31/1995} \) and 90% of \( VE_{12/31/1995} \) as the most likely value from the distribution of values determined from the simulation of value based on your extended valuation (3) above, using the same terminal year (this is identified as "Jan's seventh task" on page 523).

For the simulation, vary the following forecast parameter variables within sensible ranges, and report the most likely value \( VE_{12/31/1995} \) of RMAG selected from your simulation results. For each forecast parameter, sketch the distribution you think likely for the numerical values of the forecast parameter within its minimum-maximum range. Choose values from this range as inputs to your simulation.

- a) COGS/Sales, a fraction
- b) R&D/Sales, a fraction
- c) SG&A/Sales, a fraction
- d) Cancer Diagnostics Revenue, in dollars
- e) Agriculture Revenue, in dollars
- f) \( g_{\infty} \) used in Terminal Value, a fraction
- g) \( k_e \), a fraction

Turn in one spreadsheet each showing each individual valuation (1), (2), and (3), and at least 18 spreadsheets for the simulation (4). In each spreadsheet, give the value of each of the variables a – g listed above, and any other forecast parameters and assumptions required to generate the spreadsheet numbers.

These three forecasts differ in the location of the forecast horizon. The "forecast horizon" is the last year which is explicitly forecast, the location of the Terminal Value:

1) RMAG sets the forecast horizon at 2005 (See Exhibit 1);
2) Big Sur sets the forecast horizon at 2006 (See Exhibit 2);
3) and 4) you set the forecast horizon at the year you predict RMAG will become a mature firm.
Use the Gordon model to compute the Terminal Value. In each forecast, the Terminal Value of the firm must be created and located at the year of the forecast horizon. The Terminal Value is discounted along with all the annual cash flows back to the present to compute the present value of equity.

There are also subsidiary tasks you must accomplish and report; they are identified in the case texts as "Task n". Two of the text's Tasks are omitted (No. 2 and "Kate's Task"), and I have changed Task 3 and Task 6. Task 7 is your fourth valuation using the simulation. You will be another assistant to Kate McGraw (not Janice Kelley) writing a report to Kate McGraw.

You must value the equity of the entire firm Rocky Mountain Advanced Genome, Inc. (RMAG) and 90% of the value of the equity. Report the value of equity and 90% of that value.

Use both the RMAG and Big Sur forecasts (note the different number of years of the two forecasts); add a terminal value to each for which you choose, identify, and explain a perpetual growth rate \( g_\infty \); discount the future leveraged free cash flows to equity at \( k_e = 20\% \) to present value \( VE_0 \); and report the value of RMAG by both forecasts. Then take 90% of each value and report the initial bargaining range based on this portion of the equity. Your report must begin with a summary section reporting these values. After these two forecasts, running through 2005 (for RMAG’s forecasts) and 2006 (for Big Sur’s forecasts), you must make a third forecast, adding years of cash flow forecasts to whichever base valuation scheme you prefer, out to the year you select for the Terminal Value, when you predict RMAG will become a mature firm. Then take 90% of the value of equity of this forecast you have prepared, and report it as well. After running these three single-value valuations, use the third forecast as the basis for a fourth valuation, in which you simulate the value of RMAG by altering each of the identified input variables within its appropriate range (which you must specify), and then choosing the mean value of the resulting distribution of values as your estimate of the "value of RMAG".

You will use the method of discounting future leveraged free cash flows to equity to present value using the cost of equity capital given as \( k_e = 20\% \), in Valuations 1, 2, and 3. In Valuation 4, the simulation, you will vary several parameters, including \( k_e \) and \( g_\infty \).

As part of your report, turn in:
1) one spreadsheet showing this valuation using the RMAG forecasts,
2) one spreadsheet showing this valuation using the Big Sur Forecasts,
3) one spreadsheet showing the valuation using your extension of one of the forecasts to a more appropriate forecast horizon date, and
4) at least 18 spreadsheets, each simulating the value of RMAG using the horizon date of Part (3);
   on each spreadsheet, show the vector of values of the forecast parameters for that run of the simulation.

You may also prepare other spreadsheets using different sets of assumptions, if you wish.
5) a distribution of the value of equity computed by the simulation showing your selection of the most likely value in the "histogram" distribution.

\[
VE_0 = \sum_{t=1}^{T} \frac{LFCFE_t}{(1 + k_e)^t} + \frac{TV_T}{(1 + k_e)^T}
\]

where

\[
TV_T = LFCFE_T \frac{(1 + g_\infty)}{(k_e - g_\infty)}
\]
This discounting procedure gives the value of equity now $V_{E0}$, and the value of equity now is the relevant measure of value, since Big Sur Capital Management Company will be purchasing 90% of the equity of Rocky Mountain Advanced Genome. The management of RMAG suggests a price of $46 million for 90% of the equity. You must evaluate the firm to see whether this is the proper price or not. RMAG has no debt, so its weighted-average cost of capital is equal to the cost of equity. The discount rate is given as $ke = 20\%$. Because RMAG has no debt, it is "unleveraged", and the computation of the leveraged free cash flows to equity is thereby simplified somewhat.

You must choose a future perpetual growth rate $g_{\infty}$ beyond the explicit forecast years 1 to $T$, ending with $T$, with which to compute the Terminal Value at time $T$, ("TV$_T$") using the Gordon Constant Perpetual Growth Model. You may use a different perpetual growth rate $g_{\infty}$ for each set of future cash flows: those of RMAG shown in Exhibit 1, those of Big Sur shown in Exhibit 2, and the forecast you make for the longer period to your forecast horizon. The value you use for $g_{\infty}$ need not be the same as that of the last explicit forecasted year; explain in each case why you chose the value you did for $g_{\infty}$. In your simulation, you will vary $g_{\infty}$ to provide a range of Terminal Values.

$$TV_T = \frac{LFCFE_T (1 + g_{\infty})}{ke - g_{\infty}}$$

Two concepts which you must think deeply about are:
1) the duration of the explicit forecasts of cash flows; i.e., the number of years 1 to $T$ of explicit forecasts prior to the assumption of stability and constant subsequent growth (which is the same as deciding where to place the terminal value at time $T$); and
2) how to take account of "forecasting risk": the unavoidable errors in forecasting future dimensions, or variables, of the firm (such as the ratio of cost of goods sold to sales, or the ratio of administrative expenses to sales, or the fixed asset turnover, or the growth rate of sales) and of the economy (such as the inflation rate, the term structure of interest rates, or the market price of risk). Changes in these and the other variables will affect the value which you calculate; i.e., if the actual size of a variable which occurs is different from the size you forecasted, the actual value of the firm will be different from your "point forecast." This is forecasting error and it reflects forecasting risk, and it must be addressed in a competent valuation.

There are three techniques available for evaluating this forecasting risk:

a) sensitivity analysis, in which one variable at a time is changed, and the effects of these changes on the present value of the firm are observed; one reports a range of values dependent on the size of one input variable; this is the technique used in "Data Tables" or "Sensitivity Analysis Tables";

b) scenario analysis, in which a group of few variables forming a "scenario" is altered a few times, (five to ten different scenarios are commonly used) with each variable in the vector taking on a new value in each scenario (i.e., all values are changed for each scenario run), thereby creating a few "scenarios," and the effects of these simultaneous changes in different variables on the value of the firm are observed; one reports a range of values dependent on different combinations of different sizes of a few input variables; and

c) simulation analysis, in which all (or at least a number) of the forecast variables or "parameters" are each independently altered according to a reasonable possible distribution of each individual variable within their reasonable limits, and the value of the asset is computed with each set of values of the input variables, many, many, many times as the combination of values changes; a distribution of present values results, and the mean value of this distribution is identified as the best estimate of the value. A simulation is just like many, many scenario analyses repeated many, many times, so that instead of five or ten runs, hundreds are made.

Simulation analysis takes account of the independent variability of the forecast variables (e.g., the ratio of administrative expenses to sales is presumably independent of the rate of inflation, and the operating expenses to sales ratio may be independent of the capital structure ratio) and the effect on the computed value of the firm of these variabilities as they interact.

You must discuss Forecasting Risk Analysis in a separately-titled section of your report regarding Valuation (4) using simulation.

FORECASTS OF FUTURE CASH FLOWS:
Both RMAG and Big Sur have each individually computed forecasts of free cash flows specifically for each of the next ten or eleven years, 1996-2005 and 1996-2006, and these two different specific cash flow forecasts are given in Exhibit 1 as developed by RMAG, and in Exhibit 2 as developed by Big Sur. Note that RMAG forecasted through 2005 and Big Sur forecasted through 2006. You may have to add several years of forecasted cash flows to these forecasts to reach the time you select for the terminal value in your forecast valuation.

You will have yet a third period of discounting for your own horizon date. Compute three different single-point values of the firm (Parts 1, 2, 3), and compute the a few (1 to 3) percentage points around 20%. Notice that the periods of discounting are different for the two forecasts, and use a discount rate of 20%, as given for (1), (2), and (3); for the simulation (4), you will be varying the discount rate.

You must forecast the terminal value at the horizon date (i.e., at the end of the explicit years forecasted) for each forecast cash flow sequence:

Presumably, RMAG’s evaluation of the firm’s equity as being worth $51.11 million (so that 90% is worth $46 million) comes from their cash flow forecast. But they have not specified a terminal value, a perpetual growth rate, or a discount rate. Use ke = 20% as the discount rate. You should forecast a terminal value TV_{2005} for the RMAG cash flow forecast (and to do this, you must forecast a perpetual growth rate g_{\infty} for the years after 2005), and discount all (cash flows and terminal value) back to present value VE_0 (RMAG) to check their valuations. Remember that

$$TV_{2005} = \frac{LFCFE_{2005} \times (1 + g_{\infty})}{(ke - g_{\infty})}.$$

Then, using the cash flow forecasts of Big Sur, also forecast a terminal value TV_{2006} for the Big Sur valuation scheme and discount all (cash flows and terminal value) back to present value VE_0 (Big Sur) to see what your firm Big Sur thinks RMAG is worth.

$$TV_{2006} = \frac{LFCFE_{2006} \times (1 + g_{\infty})}{(ke - g_{\infty})}.$$

Forecast a Terminal Value at T, the horizon date you have chosen for your forecast and valuation. Then, using the cash flow forecasts you choose as more accurate, discount all (cash flows and terminal value) back to present value VE_0 (Your Name) to see what you think RMAG is worth.

$$TV_T = \frac{LFCFE_T \times (1 + g_{\infty})}{(ke - g_{\infty})}.$$

Use a discount rate of 20%, as given for (1), (2), and (3); for the simulation (4), you will be varying the discount rate a few (1 to 3) percentage points around 20%. Notice that the periods of discounting are different for the two forecasts, and you will have yet a third period of discounting for your own horizon date.

Compute three different single-point values of the firm (Parts 1, 2, 3), and compute the most likely value of the firm from the distribution resulting from the simulation (Part 4):

Use each of these two given sets of cash flow forecasts (RMAG’s (1) and Big Sur’s (2)) and your own forecasted set of cash flows (3) based on either RMAG’s or Big Sur’s, to which you extend the cash flows for a number of years that you choose to get to the terminal year you choose to determine the value of the firm and report the results of each forecast. To do this, you will have to identify three values of the terminal value, one for each cash flow forecast (RMAG’s, Big Sur’s, and yours), and you will also have three different horizon dates, and possibly three different perpetual growth rates for the future period beyond either 2005 or 2006 (one for RMAG’s forecast and one for Big Sur’s forecast) and your own horizon date. You will have three explicit values of the firm for the one-point forecasts, and a simulation with an expected value of the firm for the simulated forecast in (4); one based on RMAG forecasts, incorporating its associated terminal value and perpetual growth rate; the second one based on the Big Sur forecasts, incorporating its associated terminal value and perpetual growth rate; the third one based on your own forecasts of annual cash flows, perpetual growth beyond the horizon date, and your Terminal Value; and the fourth one based on (3) but with simulations. Use the same 20% discount rate for each value computation in (1), (2), and (3), and alter ke along with the other forecast variables I specify to make the simulation (4). Then you can multiply each value by 90% to deduce the proposed exchange value from the point of view of each forecast.

Use the free cash flows and horizon date you have chosen for valuation (3) and vary the specified parameters at least through “low”, “medium”, and “high” values each, to produce a series of simulated values of the firm. Also vary the ke around 20%. Then report the most likely value of this distribution of values as the “value” of RMAG.

Free Cash Flow to Equity = LFCFE:

Free cash flow is the following (see the bottom of Exhibit 1 and the bottom of Exhibit 2. Remember I gave you a clarification of Free Cash Flow. The firm RMAG has no debt, so there are no adjustments needed for principal payments or new debt capital, and you can neglect preferred dividends and new equity capital.

\[
LFCFE_t = NIAT_t + DEPRECA_{t} + AMORT_t + IVS_t - \Delta NWC_t
\]

Free Cash Flow = NIAT + Depreciation + Amortization + Other non-cash expenses
(and - any non-cash revenues) - Increases in Working Capital
(or + Decreases in Working Capital) - New Capital Investments
IN ADDITION TO VALUING RMAG, PLEASE ALSO PERFORM TASKS 1, 3 (altered), 4, 5, and 6 (altered) WHICH ARE INVOLVED WITH THE ESTIMATION OF THE TERMINAL VALUE, AND WRITE A SECTION ABOUT EACH TASK:

In addition to valuing the entire firm and 90% of RMAG as discussed above, also perform the tasks listed in boxes throughout the case text, as modified by the instructions below, and write explicitly about each. Your conclusion is the value of the firm Rocky Mountain Advanced Genome, Inc., and what your firm Big Sur Capital Management should pay for 90% of the equity. Show explicitly on your spreadsheet the present value equation, including the terminal value, which is used to compute the value of the firm. In addition, write a separate section of your report for each of the "Tasks" I ask you to do. Please perform Tasks No. 1, 3, 4, 5, 6 only. You are already performing Task 7 in your fourth valuation by simulation. Note that I have altered both Task 3 and Task 6. Please do not do Task 2. Please do not do Kate's Task as given in the "Conclusion" on page 523.

Your first task is to explain the data of Exhibit 3 regarding the portion of present value explained by the specifically-forecasted cash flows and the terminal value.

Ignore the stated second task to evaluate the quaint "alternative measures" of terminal value in Exhibit 4. In my opinion, the discounted cash flow measure, discussed in these notes, is the best method to estimate the terminal value. In my opinion, book value, multiples methods, and other methods all omit critical variables. Some people disagree with me, but they are wrong. "Multiples" methods sometimes give useful information, but you need not use them in this case solution, despite their ease of use.

TAXES ARE CONSIDERED IN THE LFCFE NUMBER; DO NOT DOUBLE-TAX:

The cash flow from operations begins with Net Income After Taxes, so the impact of income taxes is already considered at the beginning of the Free Cash Flow forecast. Therefore, since the terminal value is computed on the basis of the cash flow net of income taxes, you should ignore income taxes on the supposed gain on the terminal value, because you would otherwise be counting taxes twice.

DO NOT CONSIDER "LIQUIDATION"; THIS IS A GOING CONCERN:

Do not get confused by the ignorant silly girl's mention of liquidation value as a possible way to estimate terminal value.
USE OF THE CONSTANT GROWTH (GORDON) MODEL FOR THE TERMINAL VALUE:

Do not use market multiples to estimate the terminal value; use the Gordon model. This is correct if you have correctly estimated when the firm will reach maturity and its constant perpetual growth will begin.

THE PERPETUAL GROWTH RATE MUST BE SMALLER THAN THE DISCOUNT RATE:

Note that the discount rate must always be larger than the growth rate \( (k_e > \bar{g}) \) in order for the Gordon-model to compute a finite terminal value. The mathematics require that the discount rate must be greater than the growth rate; the economics do too.

IN THE GORDON CONSTANT-PERPETUAL GROWTH MODEL USED TO COMPUTE THE TERMINAL VALUE \( TV_T \):

THE PERPETUAL GROWTH RATE \( \bar{g} \) (BEGINNING AT THE FORECAST HORIZON AND FOLLOWING THE FORECAST HORIZON FOREVER) MUST BE SMALLER THAN HISTORICAL GROWTH RATES PRIOR TO THE FORECAST HORIZON, AND IT MUST ALSO BE SMALLER THAN THE DISCOUNT RATE USED TO COMPUTE THE TERMINAL VALUE:

STATE EXPlicitly THE ASSUMED NUMERICAL VALUE FOR PERPETUAL GROWTH RATE \( \bar{g} \):

For the third task, DO NOT bother with the setting of the specific horizon date and location of the terminal value of the three example firms of Exhibit 5. (In the case of the toll road, the terminal value could be estimated in year 3; for the bottling plant, year 11; for the film studio, year 28.) Rather, set the specific horizon date and the location of the terminal value for RMAG, and use this in your valuation, and explain how you chose the horizon date. If you choose a horizon date different from either 2005 or 2006 be sure to forecast additional years of cash flows to go out to the horizon date. And be sure that you locate the Terminal Value at the horizon date you select.

In choosing the horizon date and the implied perpetual growth rate beyond the horizon date, the key idea is that the perpetuity estimators—the constant perpetual growth rate and the cash flow of the firm at the start of the perpetuity—require an assumption of maturity of the business; that is, in this constant and stable existence, the firm is mature and growing at a constant perpetual (and small) rate. The firm should have completed the high-growth phase of its existence. It would be wrong to use a multiple from a young firm (such as we see today in the comparison firms discussed) in valuing the mature firm which RMAG will become in ten to fifteen years or so. Write a detailed explanation in your case solution of how you determined the horizon date and why you think that is the end of the unstable or rapid-growth portion of the firm's lifetime, or alternatively, why you have placed the terminal value where you have in your third valuation and the simulation. If you think the firm will not reach stable maturity until beyond 2005 or 2006, you must forecast the required additional years of cash flows to go out to the year of the maturity and terminal value.
TERMINAL VALUE USING THE GORDON MODEL:

Recall the valuation of a share of stock: you discount the dividends to be received in each year you will own the stock, and add to that the price you will receive from selling the stock after you receive the last dividend. Suppose you plan to hold the stock for five years and sell it after receiving the fifth dividend. Then the value of the stock to you is the following:

\[
V_0 = P_0 = PV(d_1) + PV(d_2) + PV(d_3) + PV(d_4) + PV(d_5) + PV(P_5)
\]

\[d_t = \text{the dividend received at time } t\]

\[PV(d_t) = \text{the present value now of the dividend to be received at time } t\]

where PV indicates the present value of a future cash flow today, achieved by discounting that future cash flow at \(ke\), the cost of equity capital (the rate of return required on this equity investment); for example,

\[PV(d_t) = d_t / (1 + ke)^t\]

and

\[PV(P_5) = P_5 / (1 + ke)^5\]

here \(ke\) = the cost of equity capital, the rate of return required to be earned on the equity investment;

\[P_5 = \text{the selling price of the stock at time } t_5; \text{ this is the value at time } t_5 \text{ of all of the subsequent dividends (beyond } t_5, \text{ so beginning at } t_6) \text{ which the stock will pay for the remainder of the lifetime of its issuing firm; the first dividend included in } P_5 \text{ is the dividend at time } t_5; \text{ so the price at time } t_5 \text{ is the "present value" at } t_5 \text{ of all the dividends to be paid in the future beyond } t_5, \text{ that is at } t_6, t_7, t_8, t_9, \text{ and so on to the end of the lifetime of the firm, whenever that is. Notice that this price } P_5 \text{ at which you assume you will sell the stock at } t_5 \text{ is the present value at } t_5 \text{ of the dividends beginning one year after } t_5. \text{ } P_5 \text{ is the present value at } t_5 \text{ of the many-year stream of dividends from } t_6 \text{ to infinity; if the growth rate from } t_6 \text{ to } t_\infty \text{ is constant, then } P_5 \text{ can be expressed using the Gordon model with the dividend at time } 6:\]

\[P_5 = d_6/(1+ke)^4 + d_7/(1+ke)^5 + d_8/(1+ke)^6 + ... + d_\infty/(1+ke)^\infty = d_6 / (ke - g)\]

The complete present value today at \(t_0\); i.e., \(P_0\), is then the sum of the present values of the dividends from \(t_1\) through \(t_5\) plus the present value of \(P_5\):

\[P_0 = V_0 = d_1/(1+ke)^1 + d_2/(1+ke)^2 + d_3/(1+ke)^3 + d_4/(1+ke)^4 + [d_5 + P_5]/(1+ke)^5\]

Most commonly, the value of \(P_5\) is computed using the Gordon constant perpetual growth model, assuming that the dividends \(after d_t\) will grow forever at a constant rate \(g\), and computing the value of \(P_5\) using \(d_t\) and the subsequent growth rate. Note that \(d_t = d_t X (1 + g)\), so the constant-growth process begins at \(t_5\) and goes on from there. The price is therefore computed at the moment when the constant growth process begins, \textit{after} the changing-growth-rate period, or the faster-growth-rate period (which ran from \(t_0\) to \(t_5\), ends.

Usually, \(P_5 = d_5/(ke - g)\). Use of the Gordon model for the selling price is a convenient simplification only.

In this stock valuation model, the first five dividends are \textbf{specifically forecast}, and the remaining dividends are \textbf{not specifically forecast}, but are included in the valuation by means of the selling price \(P_5\), often called the "\textbf{terminal value}" or "residual value". You can consider the selling price \(P_5\) as the \textbf{terminal value} of the stock to you: the value it has at the end (terminus) of the period you have specifically forecast.

\textbf{Note that the terminal value occurs one period before the first cash flow which is included in the terminal value: }TV_T \text{ depends on } FCF_{T+1}. \text{ Note that the terminal value is also discounted back to the present to form a portion of the present value.}

In computing the value of the stock to you today (the present price \(P_0\) or the "value" or "present value" \(V_0\)), you discount the five specifically forecasted dividends, \textbf{and you also discount to today the terminal value }\(P_5\), \textbf{which you expect to receive at }\(t_5\) \textbf{when you sell the stock after receiving the dividend }d_5. \textbf{ }(\textit{ }P_5\textit{ is of course the present value at }t_5\textbf{ of all the...}
subsequent dividends to be received, from \( t_6 \) out to \( \infty \). This second component of the value of the stock is the present value of the selling price \( P_5 \), or the present value of the terminal value \( T_5 \).

The "terminal value" \( V_5 \) or \( T_5 \), is the value at the time of the last year specifically forecast; i.e., year no. 5, of the cash flows of the firm beyond the year which is last specifically forecast. The terminal value is like \( P_5 \), the price at \( t_5 \) in the stock example above. The "present value of the terminal value" is the value today at \( t_0 \) of that \( t_5 \) terminal value.

\[
V_5 = \frac{CF_6}{(1+k)^1} + \frac{CF_7}{(1+k)^2} + \frac{CF_8}{(1+k)^3} + \ldots + \frac{CF_N}{(1+k)^N}
\]

And \([PV_0 \text{ of } V_5] = V_5/(1+k)^5\)

For example, if you are valuing a firm with a lifetime of fifty years beyond today, and you specifically forecast five years of future cash flows to a horizon date five years from now, then the remaining forty-five years of lifetime beyond the horizon date are not specifically forecast. But those forty-five years of remaining lifetime beyond the five years you forecast (that is, the lifetime from year 6 to year 50) contribute value—a great deal of value—to the present value of the firm. The terminal value is the value at time five of those remaining forty-five years of lifetime, so you compute the value at time five of the cash flows from time six through time fifty, a forty-five year span of unequal cash flows. The present value of the terminal value is the value today at time zero of the terminal value, obtained by discounting the terminal value back to the present moment. So to get the present value of the terminal value, you must discount the terminal value (which exists in year 5) back to present value today.

\[
V_0 = d_6/(1+ke)^1 + d_7/(1+ke)^2 + d_8/(1+ke)^3 + \ldots + d_{45}/(1+ke)^{45} + V_5/(1+ke)^5
\]

Terminal value is usually between 60% and 90% (or even more, depending on the number of years specifically and explicitly forecast) of the total lifetime of the firm, and on the shape of the cash-flow stream of the present value of an enterprise that is being valued because the specific number of years being forecasted is small and thereby neglects much of the lifetime growth and value creation of the firm. The terminal value lacks precision in forecasting and analysis, so it effectively hides the specific information assumed about the future; the larger the portion of the present value contributed by the terminal value, the greater the portion of the present value which remains unknown and undefined. It is virtuous to reduce the portion of the present value contributed by the terminal value because this clarifies the specific forecasts and hides less of the forecast under a blanket.

The greater the number of years of specific prediction and explicit cash flow estimation, the smaller is the portion of the present value of the firm hidden in the terminal value; the smaller the number of years of explicit forecast, the greater is the portion of the present value hidden in the terminal value. The number of years of specific prediction when most people are doing the valuation is usually three to five; some people go out to ten years: hence, when these people value a business, the terminal value (which hides all the variables into one cash flow and a constant growth rate) contributes 70% to 90% of the total value of the firm, thereby hiding the greatest source of value. However, if the number of specific years being explicitly and specifically forecasted (using the detailed forecast values) were larger, say ten, fifteen, twenty, or twenty five, then the terminal value would be a smaller and smaller percentage of the total value, and this "hidden source of value" would not contribute the overwhelming majority portion of the total value. If you forecasted to the very end of the firm's lifetime, or effectively for thirty to fifty years explicitly, there would be no or a negligible terminal value, and the terminal value would contribute nothing whatever to the value of the firm.

The terminal value should be removed to the future beyond the period of rapid or changing growth, to the start of the equilibrium period of firm maturity, where stable operation will occur, with growth from year to year in sales equal to the inflation rate, or the rate of growth of population, and with market share constant. "A key point of judgment in valuation analysis is to set the forecast horizon at that point in the future where stability or stable growth begins." (page 520, emphasis in original). And the growth rate, assumed to be constant most often in terminal value calculations, cannot be very large because if it were greater than the population growth plus inflation, the firm would expand market share each year, and soon be the entire industry, and soon after that, be the entire economy.

Read the discussion of when stability sets in for the three example firms on pages 520, to see where the terminal value should be placed, after the firm reaches stability.

**TERMINAL VALUE FOR RMAG REQUIRES MORE IVS OUTLAYS THE LARGER THE PERPETUAL GROWTH RATE.**

**SUSTAINABLE GROWTH: FASTER GROWTH NEEDS MORE ASSETS, BOTH LONG-TERM CAPITAL ASSETS AND WORKING CAPITAL.**
FASTER GROWTH REQUIRES LARGER IVS EXPENDITURES AND LARGER NWC OUTLAYS.

**Task Four** is to Interpret Exhibit 6, which presents a sensitivity analysis of the effect of the perpetual growth rate and the various assumptions which contribute to the terminal value, on the present value of the firm, for each of the two preparations (RMAG and Big Sur). **Write a DETAILED AND SPECIFIC analysis of this table.** Compare the capital expenditures shown in Exhibit 6 for 2006 (RMAG’s View) or 2008 (Big Sur’s View) with the capital expenditures shown in Exhibit 1 or 2, respectively, for 2005 or 2006. Note that larger growth rates in Exhibit 6 have larger expenditures for capital goods and larger additions to Net Working Capital. Recall the Sustainable Growth Rate analysis.

**Task Five** is to explain the causes of the numerical value of the perpetual growth rate which contributes to the terminal value. A paragraph or so should suffice.

The **sixth task** is defined on page 523 to estimate terminal values using alternative approaches. **Do not do this: merely estimate the terminal value using the Gordon constant-growth method.** Then, having determined the terminal value, comment on the "price multiple," $TV_T/CF_T$, which it exhibits at the point where it occurs, and comment about where in the range discussed on page 523 this multiple lies.

"MARKET MULTIPLES AND CONSTANT GROWTH VALUATION":

The discussion on pp. 518-519 essentially derives the Gordon constant-perpetual growth model. If the firm has debt in the capital structure, then use the unlevered free cash flows, and discount them by the weighted-average cost of capital. This gives you the value of the entire firm, from which you must subtract the value of debt to obtain the value of equity.

If the firm has no debt in the capital structure so is an all-equity firm like RMAG, then use the leveraged free cash flows to equity (called "residual cash flow, RCF" on page 519) and discount them at the cost of equity capital, which will produce the present value of equity directly. Note that RMAG has no debt in its capital structure, so the entire free cash flow is available to equity suppliers. The discount rate of $ke = 20\%$ is appropriate in this circumstance (see page 522 in the section on "Terminal Value").

Note also that $k > g$ always: the discount rate must be larger than the growth rate, so long as the firm ever generates cash flows.

Write a paragraph in your report giving your forecast of the constant perpetual growth rate $g_\infty$, which RMAG will likely have during the years following the specific forecast period, and which you used in your computation of terminal value. Be sure to discuss real growth and the expected inflation rate. The current rate of growth of the U.S. population is about 0.5% per year. Currently, inflation in the United States is about 2.5% per year and is expected to average between 1% and 4% per year over the foreseeable future. The recent real growth rate in pharmaceutical revenues has been 3% to 6% per year. You must state the value of the long-term (perpetual) constant growth rate you use.

Recall from our discussion of Sustainable Growth that, if the operating efficiency of the firm remains constant, the only way to achieve growth is to increase the assets of the firm or operate them longer each day. So faster growth requires more investment in new fixed assets and new working capital assets to sustain the higher growth rate. This is an interdependence between the growth rate and the cash flows of the firm. You must take this interdependence into account in your cash flow analysis and growth rate forecast.

Note the exercise in Exhibit 6 on page 529 computing the value of the firm for different values of the growth rate for both sets of forecasts of cash flows. Note that each different growth rate requires different amounts of new capital expenditures and new working capital expenditures, as shown in the table at the bottom of page 522. Recall that faster growth requires more assets if the operating efficiency of the assets is not increased. This is the **fourth task**, as identified on page 522, and the **fifth task**, as identified on page 522.

The **seventh task** is simulation of the value of RMAG taking account of forecasting risk, as discussed above, and is what you have already done in the fourth valuation exercise. In your simulation on the cash flow forecasts, which used a range of discount rates, you generated a distribution of possible values, under each set of growth-rate assumptions. You then simulated the growth rates, and you got an overall distribution of values, from which you could select the middle value as most likely. This is, of course, the best way to perform a valuation: to take explicit account of the risk in each variable and the possible variation in firm parameters and economic variables, and see how the possible variations in each factor during the future interact together to influence the value in the present.

**Do not worry about Kate’s task given on page 523.**
Summary

A.) Now you are going to use these two forecasted pro-forma statements to calculate the Leveraged Free Cash Flows to Equity \( LFCFE_t \), and the Unleveraged Free Cash Flows to the Firm \( UFCFF_t \), for the next two years, compute a Terminal Value for each, based on the appropriate cash flow and your assumed long-term growth rate of that cash flow, and discount each stream to present value using \( ke \) and \( kf^* \), respectively; \( i.e. \), using \( ke \) to discount the \( LFCFE_t \)'s and their TV to \( VE_0 \), and using \( kf^* \) to discount the \( UFCFF_t \)'s and their TV to \( VF_0 \). Then you will compare the two computations of \( VE_0 \), the first using the discounted stream of \( LFCFE_t \)'s and their TV, and the second as \( VE_0 = VF_0 - VD_0 \).

B.) Next, you will identify the ranges of forecasting error likely for each forecasted number used to construct your pro-forma statements (the "forecast parameters"), you will discuss the likely shape of the distribution of the range of possible alternative values of each forecast parameter, and you will postulate the distribution of each forecast parameter within its boundaries according to the shape you have identified.

C.) After that, you will use the distributions of the forecast parameters to simulate the value of the firm \( i.e., \) compute many different values of equity or values of the firm, each present value of equity or value of the firm computed using a vector of different numerical values of each forecast parameter), using the two methods; \( i.e. \), \( VF_0 = PVAL(UFCFF + TV) \) and \( VE_0 = PVAL (LFCFE + TV) \). Select the most likely result from the distribution of values, as the "value".

Greater Detail:

A. Value your Firm; \( i.e., \) compute one only \( VE_0 \) for both methods:
1) discount the \( LFCFE_t \)'s and TV using \( ke \) to determine \( VE_0 \);
2) discount the \( UFCFF_t \)'s and TV using \( kf^* \) to determine \( VF_0 \), then subtract \( VD_0 \) from \( VF_0 \) to get \( VE_0 \).
Value your Firm; i.e., compute $VE_0$ for both methods.

Compute the Leveraged Free Cash Flows to Equity and the Unleveraged Free Cash Flows to the Firm from your pro-forma statements for $t_1$ and $t_2$. Use only the most likely value of each forecasted parameter; i.e., make only one forecast for the two years. Use the Gordon model to compute the Terminal Value at $t_2$ for each CF stream. Insert the Terminal Value into the cash flow stream, estimate the cost of equity, estimate the cost of debt, estimate the weighted-average cost of capital. Compute the value of the firm using both methods: 1) discount the LFCFE's and the TV using $ke$; and 2) discount the UFCFF's and the TV using $kf^*$. Discuss any differences. Compare your computed values with the actual market value of the equity of the firm and the total market value of the firm, and with the book value of the total assets and with the book value of equity. Explain any differences. Discuss your valuation procedures and what you have learned.

Compute the Leveraged Free Cash Flows to Equity and the Unleveraged Free Cash Flows to the Firm from your pro-forma statements for $t_1$ and $t_2$. Use only the most likely value of each forecasted parameter; i.e., make only one forecast for the two years. Use the Gordon model to compute the Terminal Value at $t_2$ for each CF stream. Insert the Terminal Value into the cash flow stream, estimate the cost of equity, estimate the cost of debt, estimate the weighted-average cost of capital. Compute the value of the firm using both methods: 1) discount the LFCFE's and the TV using $ke$; and 2) discount the UFCFF's and the TV using $kf^*$. $ke$ is determined by the Capital Asset Pricing Model: $ke = R_f + \beta (E[R_M] - R_f)$. If another risk adjustment is necessary, it must be added.

The value of the firm $VF_0$ is the present value to the owners (debt holders and equity holders) of all the future benefits they will receive from the firm for the remainder of its lifetime forward from the present day. The value of equity $VE_0$ is the value of the firm minus the value of debt: $VE_0 = VF_0 - VD_0$. The value of equity $VE_0$ is also the present value to the equity holders of all the future net benefits (dividends minus new equity capital supplied) they will receive from the firm for the remainder of its lifetime forward from the present day.

We calculate the Value of Equity $VE_0$ by discounting the series of the "Leveraged Free Cash Flows to Equity at each time point" (LFCFE$_t$) from $t=1$ through $t=T$ plus the Terminal Value at $T$, $TV_T$, all by the cost of equity capital $ke$. In this Problem, $t_1 = t_1$, and $T$ is $t_2$.

Free Cash Flow (FCF) or "Free Cash Flow to Equity" (FCFE) or "Leveraged Free Cash Flow to Equity" (LFCFE) is the annual cash flow the firm can use to pay common dividends after the company has made all the investments in fixed assets and working capital necessary to sustain ongoing operations and without adversely affecting the planned growth of the firm; i.e., after purchasing plant and equipment needed to maintain operations and achieve growth, and after purchasing the net working capital required to operate the new plant and equipment. This definition of "Free Cash Flow" subtracts the required debt service ( PP + I ) for the year on the way to the computation of FCF. We call this "Leveraged Free Cash Flow to Equity Suppliers", LFCFE. (Stickney)

$$VE_0 = \sum_{t=1}^{t=T} \left[ \frac{LFCFE_t}{1 + ke} \right] + TV_T / (1 + ke)^T$$

**TERMINAL VALUE** at time $T = TV_T$:

$TV_T = Terminal Value_T$ = the value at a particular time point $T$ of all of the following cash flows for the remaining lifetime of the firm; i.e., beginning at $T+1$ and going on to $\infty$. The value
of a future cash flow occurring at some future time point, valued at a particular time point T, is
computed by discounting that cash flow from its time point of occurrence back to the particular
time point T at the proper discount rate or "cost of capital". All of the present values at time T of
all of the future cash flows are then added together to get $TV_T$.

$$TV_T = \text{Terminal Value at time } T = (\text{Relevant Cash Flow}_{T+1} ) / (ke - g_\infty )$$

From the Gordon Constant-perpetual-growth model,

where

- Cash Flow$_{T+1} = $ Cash Flow$_T \times (1 + g_\infty )$
- $k = $ the proper discount rate, ke from the Capital Asset Pricing Model, usually;
- $g_\infty = $ the constant perpetual growth rate from time T to infinity.

The relevant cash flow is the same cash flow as was discounted explicitly at prior t's.

$LFCFE_t = \text{Leveraged Free Cash Flow to Equity at time } t$

The stream of $LFCFE_t$'s plus $TV_T$ is discounted at $ke$ to compute $VE_0$.

$LFCFE$ can be used to pay common dividends or repurchase Treasury stock.

In any year $t$,

$$LFCFE_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t - \text{IVS}_t - \Delta \text{NWC}_t - \text{PP}_t + \text{NDC}_t + \text{NPS}_t - \text{Pfd Divs}_t$$

$LFCFE_t = \text{Leveraged Free Cash Flow to Equity}_t =$ available dividends in year $t =

= $ NIAT$_t$ + Noncash charges$_t$ - IVS$_t$ - $\Delta$NWC$_t$ + $\Delta$NDC$_t$ + NPS$_t$ - Pfd Divs$_t$

$T = $ the number of years specifically forecasted; in Problem 11.4, $T = 17$.

The interest expense + the tax effect of interest have already been subtracted from NIAT.

Net New Debt Capital$_t = \text{NNDC}_t = \text{NDC}_t - \text{PP}_t$

$LFCFE$ subtracts changes in net working capital in defining $LFCFE$ so that the only task
for $LFCFE$ is the ability to pay common dividends (or repurchase Treasury stock). $LFCFE$
subtracts changes in net working capital in defining $LFCFE$ so that the only task for $LFCFE$ is the
ability to pay common dividends (or repurchase Treasury stock).

Neglecting preferred stock, we have:

Leveraged Free Cash Flow to Equity$_t = LFCFE_t =

= $ NIAT$_t$ + Noncash charges$_t$ - $\Delta$NWC$_t$ - IVS$_t$ + $\Delta$NDC$_t$ = ability to pay common dividends

where Net New Debt Capital$_t = \text{NNDC}_t = \text{NDC}_t - \text{PP}_t$

$\text{NWC} = \text{Net Working Capital} = \text{Current Assets} - \text{Current Liabilities}$

$\Delta \text{NWC} =$ the increase in Net Working Capital this year from that of last year

$\text{NPS} =$ New preferred stock

$$VE_0 = \sum_{t=1}^{T} [ LFCFE_t / (1 + ke)^t ] + TV_T / (1 + ke)^T$$

The value of the firm $VF_0$ is the present value to the owners of all the future benefits they
will receive from the firm for the remainder of its lifetime forward from the present day. If we
include "bondholders" in "owners", then the value of the debt is included in this value. If we do
not include bondholders, then the owners are only the equity holders, and the value considers only
the equity.

$\text{VALUE OF THE FIRM} = VF_0 = VD_0 + VE_0$ (assuming flat term-structure of interest rates):
VALUE OF EQUITY = VE₀ (assuming flat term-structure of interest rates):

\[ VE₀ = VF₀ - VD₀ \]

\[ VFᵣ = VDᵣ + VEᵣ + Value \ of \ Preferred \ Stockᵣ \]

0. Terminal Value \( T = TV_T \) = the value at a particular time point \( T \) of all of the following cash flows for the remaining lifetime of the firm. The value of a future cash flow occurring at some future time point, valued at a particular time point \( T \), is computed by discounting that cash flow from its time point of occurrence back to the particular time point \( T \) at the proper discount rate or "cost of capital". All of the present values at time \( T \) of all of the future cash flows are then added together to get \( TV_T \).

\[ TV_T = \text{Terminal Value at time } T = \frac{(\text{Relevant Cash Flow} \, T + 1)}{(k - g_∞)} \]

From the Gordon Constant-perpetual-growth model,

where

Cash Flow \( T+1 \) = Cash Flow \( T \) \( \times (1 + g_∞) \)

\( k = \) the proper discount rate, either \( ke \) or \( kf^* \)

\( g_∞ = \) the constant perpetual growth rate from time \( T \) to infinity.

The relevant cash flow is the same cash flow as was discounted explicitly at prior \( t \)'s.

2. \( \text{UFCFF}_t \) = Unleveraged Free Cash Flow to the Firm at time \( t \) = \( \text{UFCFF}_t \) = Cash Flow from Operations before Subtracting Cash Outflows for Interest Costs (net of tax savings) - IVS - ΔNWC + NDCᵣ

\[ \text{UFCFF}_t = \text{NIAT}_t + \text{Noncash charges}_t + \text{Interest Expense}_t \times (1 - τ) \times \text{IVS}_t - Δ\text{NWC}_t + NDC_t \]

\( \text{UFCFF}_t \) is discounted at \( kf^* \) to give the value of the firm \( VF₀ = VD₀ + VE₀ \).

\( \text{UFCFF} \) can be used to pay debt principal, pay debt interest, and pay dividends and repurchase common stock.

\( \text{UFCFF} \) does not adversely affect the firm's planned growth because the investment outlays for new plant and equipment (IVSᵣ) and the working capital to support those outlays (ΔNWCᵣ) have already been subtracted in its computation.

\[ VEᵣ = VFᵣ - VDᵣ - Value \ of \ Preferred \ Stock \]

\[ VF₀ = (t=1 \ to \ T) \sum \text{UFCFF}_t \times (1 + kf^*)^t + TV_T / (1 + kf^*)^T \]

\[ \text{UFCFF}_t = \text{NIAT}_t + \text{DepExp}_t + \text{Int}_t \times (1 - τ) - \text{IVS}_t \]

\( kf^* = \) weighted average cost of capital of the firm = \( θ \) \( kd \times (1 - τ) + (1 - θ) \times ke \)

\( θ = \) capital structure ratio of the firm = debt ratio of the firm = \( VD / VF \)

The firm should always minimize the cost of capital by reducing both business risk and financial risk.

See Stickney page 817:

NCE = Non-cash expenses; primarily depreciation and amortization.

"Unleveraged free cash flow from Operations" (UFCFO) is cash flow (that is, the non-cash expenses are added back to net income) before any payments to debtholders, either principal or interest.
(hence, it neglects financial leverage), but after income taxes (so it is "free" or "available to capital suppliers"). UFCFO = EBIT + Noncash expense – Income Tax

From that UFCFO, to get the UFCFF the unleveraged free cash flow to the firm, we must subtract the cash outlay necessary for new investment, IVS, and the new net working capital ΔNWC.

UFCFF = UFCFO – IVS - ΔNWC.

So this cash flow begins with EBIT, adds back the noncash expenses, and then subtracts the tax which was paid: NIATBI = net income after tax before interest = EBIT + NCE – Income Taxes.

Reported Cash Flow from Operations = NIAT + Noncash charges
Cash outflow for interest = Int
Interest Tax Savings = τ Int
Interest cash outflow net of income-tax savings = Int - τ Int = ( 1 - τ ) Int

Unleveraged Cash Flow from Operations

\[ \text{UFCFO}_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t + \text{Int}_t (1 - \tau) + \text{NDC}_t \]

Unleveraged Free Cash Flow to the Firm

\[ \text{UFCFF}_t = \text{NIAT}_t + \text{DEPR}_t + \text{AMORT}_t + \text{Int}_t (1 - \tau) - \text{IVS}_t - \Delta \text{NWC}_t + \text{NDC}_t \]

UFCFF can be used to pay debt principal, debt interest, dividends, and purchase common stock in the market. This is, in my view, the more useful understanding of UFCFF.

An alternative definition of UFCFF does not subtract the change in Net Working Capital so that the UFCFF can also be used to purchase new working capital. I think this is not a helpful way of looking at things, because the new working capital is needed for the planned investment outlays in new plant and equipment. But you may sometimes see this.

(Alt 2. UFCFF\textsubscript{t} = Unleveraged Free Cash Flow to the Firm at time $t = \text{UFCFF}_t = $ Cash Flow from Operations bef. Subtracting Cash Outflows for Interest Costs (net of tax savings) $\text{NIAT}_t + \text{Noncash charges} + \text{Interest Expense} (1 - \tau) + \text{NDC}_t$

UFCFF is discounted at $k^*$. 

( this definition of UFCFF can be used to pay debt principal, pay debt interest, pay dividends, and purchase new working capital.)

\[ \text{VF}_t = \text{VF}_0 - \text{VD}_t - \text{Value of Preferred Stock} \]
\[ \text{VF}_0 = (t=1 \text{ to } T)\Sigma \text{UFCFF}_t / (1 + k^*)^t + TV_T / (1 + k^* T)^T \]
\[ \text{UFCFF}_t = \text{NIAT}_t + \text{DepExp}_t + \text{Int}_t (1 - \tau) - \text{IVS}_t \]
\[ k^* = \text{weighted average cost of capital of the firm} = \theta \text{kd} (1 - \tau) + (1 - \theta) \text{ke} \]
\[ \theta = \text{capital structure ratio of the firm} = \text{debt ratio of the firm} = \text{VD/VF} \]
The firm should always minimize the cost of capital by reducing both business risk and financial risk.

NCE = Non-cash expenses; primarily depreciation and amortization.

"Unleveraged free cash flow from Operations" (UFCFO) is cash flow (that is, the non-cash expenses are added back to net income) before any payments to debtholders, either principal or interest (hence, it neglects financial leverage), but after income taxes (so it is "free" or "available to capital suppliers"). UFCFO = EBIT + Noncash expense – Income Tax

From that UFCFO, to get the UFCFF the unleveraged free cash flow to the firm, we must subtract the cash outlay necessary for new investment, IVS. UFCFF = UFCFO - IVS. This alternative
definition of UFCFF neglects the needed additional investment in new Net Working Capital to support the new investments.

So this cash flow begins with EBIT, adds back the noncash expenses, and then subtracts the tax which was paid: \( \text{NIATBI} = \text{net income after tax before interest} = \text{EBIT} + \text{NCE} – \text{Income Taxes}. \)

Reported Cash Flow from Operations = NIAT + Noncash charges
Cash outflow for interest = Int
Interest Tax Savings = \( \tau \) \( \text{Int} \)

Interest cash outflow net of income-tax savings = \( \text{Int} - \tau \text{Int} = (1 - \tau) \text{Int} \)

**Alternative Unleveraged Cash Flow from Operations**

\[ \text{Alternative Unleveraged Cash Flow from Operations}_{t} = \text{UFCFO}_{t} = \]

\[ \text{Reported Cash Flow from Ops}_{t} + \text{Int}_{t} (1-\tau) + \text{NDC}_{t} = \]

\[ \text{NIAT}_{t} + \text{DEPR}_{t} + \text{AMORT}_{t} + \text{Int}_{t} (1 - \tau) + \text{NDC}_{t}; \text{ note no } -\Delta\text{NWC} \]

**Alternative Unleveraged Free Cash Flow to the Firm**

\[ \text{Alternative Unleveraged Free Cash Flow to the Firm}_{t} = \text{UFCFF}_{t} = \]

\[ \text{UFCFO}_{t} - \text{IVS}_{t} + \text{NDC}_{t} = \]

\[ \text{NIAT}_{t} + \text{DEPR}_{t} + \text{AMORT}_{t} + \text{Int}_{t} (1 - \tau) + \text{NDC}_{t} - \text{IVS}_{t}; \text{ note no } -\Delta\text{NWC} \]

This alternative UFCFF can be used to pay debt principal, debt interest, dividends, and purchase working capital to support the new investments.

Discuss any differences between your two \( \text{VE}_0 \) computations. Compare your computed values with the actual market value of the equity of the firm and the total market value of the firm, and with the book value of the total assets and with the book value of equity. Explain any differences. Discuss your valuation procedures and what you have learned.
RANGE OF FORECASTING ERROR:

B. Define for your firm the Range and Distribution of Forecast Parameters

Define the range and distribution of each of the parameters you have forecasted to build your pro-forma income statements and balance sheets, including the growth rate of sales $g_{\text{SALES}}$, and all other forecasted parameters, such as the total asset turnover $\frac{\text{SALES}}{\text{TOTAL ASSETS}}$, the ratio of $\frac{\text{COGS}}{\text{SALES}}$, and the ratios of other operating expenses to sales; or you can specify the variable cost ratio to sales for the regression equations—choose the form of forecast you prefer: as percentage of sales or as fixed plus variable cost related to sales. You should allow the $g_{\text{OO}}$ used in the Terminal Value to vary, and you should allow the value of $ke$ to vary also; this will, of course, change the value of $kf*$. So in addition to however many forecast parameters you used to forecast the income statement and balance sheet, you should have also $ke$ and $g_{\text{OO}}$ as variable forecast parameters. The range of values of $ke$ is likely to be plus or minus one or two percentage points, and the range of values of $g_{\text{OO}}$ is likely also to be plus or minus one or two percentage points. Use your own judgment in deciding on the shape of the distribution of each forecast parameter.

For example, suppose you have a regression equation for transportation expenses as a function of sales: $\text{Transportation Expense} = 640,000 + 0.045 \times \text{Sales}$. Generally, you will keep the fixed cost value of $640,000$ unchanged and alter the variable cost value: the variable cost value is your "forecast parameter". Suppose that you believe that the minimum possible value of this slope is 0.040, and the maximum possible value of this slope is 0.050: then that is the "range of values of forecast parameters," the minimum and maximum value. Suppose that you believe that a value of 0.045 is more likely than a value of 0.040, and that a value of 0.045 is also more likely than a value of 0.050: then you have specified a centrally-peaked or near-Normal-shaped distribution with higher central tendency and symmetry. If you think the distribution is skewed, then make it skewed. If you can see no reason to prefer one value of the variable cost ratio over another, you have specified a flat distribution with equal probability of each value between 0.040 and 0.050.

You must specify for each forecast parameter, including $ke$ and $g_{\text{OO}}$, the minimum value, the maximum value, and the most likely value; you must specify the rough shape of the distribution of the possible values within the range you defined; i.e., uniform or flat, peaked, symmetric or skewed, Normal, binomial, etc. Provide a graphical picture of the distribution for each parameter, with all elements labeled: a hand-drawn sketch is fine. Explain how you defined the lower and upper limits, and explain how you selected the distribution shape and the probabilities of each value in the distribution. Explain the reason why varying the values of the forecast parameters in the forecast helps to overcome forecasting risk. The most likely value, generally the mid-point of the distribution you show, is the value you used in the valuation in part A above.

In your simulation, you will, for each calculation of $VE_0$ or $VF_0$, choose one value from the distribution of each forecast parameter, and show the listing of the selected values on the spreadsheet which determines the value. For example, if there are seven (7) different forecast parameters which you have used, then each calculation of value will begin with a vector specifying the name and the value of each of the seven parameters used in that calculation.

Define the range and distribution of each of the parameters you have forecast to build your pro-forma income statements and balance sheets, including the growth rate of sales and all forecasted parameters. You must specify for each parameter the minimum value, the maximum value, and the most likely value; you must specify the rough shape of the distribution of the possible values within the range you defined; i.e., uniform or flat, peaked, symmetric or skewed, Normal, binomial, etc. Provide a graphical picture of the distribution for each parameter, with all elements labeled. Explain how you
defined the lower and upper limits, and explain how you selected the distribution shape and the probabilities of each value in the distribution. Explain the reason why varying the values of the forecast parameters in the forecast helps to overcome forecasting risk.

**SIMULATION OF VALUE:**

C. Prepare a simulation of values of your firm by many repetitions using different values of the forecast parameters: choose the most likely value from the simulation.

Vary the discount rate also and explain your choice of range of values. Choose the most probable value of the distribution of values as the value of the firm. Prepare a full case report on the value of the firm. Compare your value with the current actual market value of the firm. Explain any difference.

Vary the discount rate also and explain your choice of range of values. You will get a number of different values of $V_{E0}$ and of $V_{F0}$ because of the different vectors of numerical values of the forecast parameters you use in the valuation computations. Include the "most likely" valuation from part A in your distribution. Sketch the empirical distribution of the value of equity and the empirical distribution of the value of the firm using reasonably-sized ranges of value: the value on the horizontal axis is the value of the firm, the value on the vertical axis is the number of occurrences in each "bucket" of value.

For example, you might run 20 simulations of the value of equity, and you might find the smallest value of equity to be $1,350,000, and the largest value of equity to be $11,430,000. You might choose the "bucket sizes" for value to be the following, and the number of valuations within each range might be as shown below. Your value of $V_{E0}$ from part A above was, suppose, $4,270,000. You now have twenty-one different values, one from part A and twenty from part C.

<table>
<thead>
<tr>
<th>Bucket Range Size</th>
<th>Number of Values in Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 - $1,500,000</td>
<td>1</td>
</tr>
<tr>
<td>$1,500,000 - $3,000,000</td>
<td>1</td>
</tr>
<tr>
<td>$3,000,000 - $4,500,000</td>
<td>4</td>
</tr>
<tr>
<td>$4,500,000 - $6,000,000</td>
<td>4</td>
</tr>
<tr>
<td>$6,000,000 - $7,500,000</td>
<td>5 the &quot;maximum likelihood&quot; value range</td>
</tr>
<tr>
<td>$7,500,000 - $9,000,000</td>
<td>2</td>
</tr>
<tr>
<td>$9,000,000 - $10,500,000</td>
<td>3</td>
</tr>
<tr>
<td>$10,500,000 - $12,000,000 +</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>21</td>
</tr>
</tbody>
</table>

There may be some buckets with no points at all, some buckets with only a couple of points, and some buckets with several points. Draw in lines connecting the points.

Choose the most probable value of the distribution of values as the value of the firm: this is the "peak" in your empirical distribution, and report the value as either the mid-point of the range or the extent of the range: e.g., "the value of the firm most likely lies between $6,000,000 and $7,500,000" or, "the value of the firm is approximately $6,750,000". Prepare a full case report on the value of the firm. Compare your value with the current actual market value of the firm. Explain any difference.
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