

Question -1. (15 points) Let  $f(x) = \cos(x)$ . Derive a formula for the  $n$ -th derivative,  $\frac{d^n}{dx^n} f(x)$  of the function. Explain how this derivative depends on  $n$  (even or odd).

Question -2. (10 points) Evaluate

$$\lim_{m \rightarrow +\infty} \left( \lim_{n \rightarrow +\infty} \frac{m}{m+n} \right) - \lim_{n \rightarrow +\infty} \left( \lim_{m \rightarrow +\infty} \frac{m}{m+n} \right)$$

Question -3. (15 points) Evaluate

$$\lim_{x \rightarrow 0} \frac{\int_0^x \frac{2}{s^3+3} ds}{x}.$$

Question -4. (15 points) Evaluate

$$\frac{d}{dx} \int_0^{x^2} e^{t^2} dt$$

*Hint:* No need to integrate. Just use the Fundamental Theorem of Calculus.

Question -5. (15 points) By using a suitable substitution show that

$$\int_0^4 \frac{f(\sqrt{x})}{\sqrt{x}} dx = 2(F(2) - F(0))$$

where  $F$  is an antiderivative of  $f$ , i.e.,  $F'(x) = f(x)$ .

Question -6. (15 points) The graph of  $y = \frac{1}{x}$  for  $1 \leq x < +\infty$  is rotated about the  $x$ -axis. Find the volume of the resulting solid.

Question -7. (15 points) (a) Find the limit of the sequence  $\{\sqrt{n+2} - \sqrt{n}\}$  if it converges.

(b) Determine whether the series  $\sum_{n=1}^{+\infty} \frac{n}{\sqrt{1+n^5}}$  is convergent or divergent. Justify your answer.

(c) Determine whether the series  $\sum_{n=1}^{+\infty} (-1)^{n+1} \frac{\ln(n^2)}{\sqrt{n}}$  is convergent or divergent. Justify your answer.