Question -1. (*15 points*) Determine numbers a and b so that $f(x) = \frac{ax}{x^2+b^2}$ has a local maximum at $x = 3$ and that $f'(0) = \frac{1}{3}$.

Question -2. (*10 points*) Evaluate

$$\lim_{x \to 0} \frac{\cos(mx) - \cos(nx)}{x^2}.$$

Question -3. (*15 points*) Evaluate

$$\lim_{x \to 0} \frac{\left(\int_0^x \sin(t)dt\right)^2}{\int_0^x \sin(t^2)dt}.$$

Question -4. (*15 points*) Show that

$$\int_0^x |t|dt = \frac{1}{2}x|x|.$$

Question -5. (*15 points*) Evaluate integral $\int_0^{+\infty} e^{-at}dt$, if it exists where $a \neq 0$. (Hint: consider case $a > 0$ and case $a < 0$ separately.)

Question -6. (*15 points*) Find the exact area of the region bounded by the graphs of $x = y^2$ and $x = 2 - y^2$.

Question -7. (*15 points*) Determine if the following sequences converge. Explain your reasoning.
(a) $a_n = \frac{1}{n^{3/2}}$.
(b) $b_n = \frac{2n}{1+n}$.
Determine if the following series converge. Explain your reasoning.
(c) $\sum_{n=1}^{+\infty} \frac{1}{n^{3/2}}$.
(d) $\sum_{n=1}^{+\infty} \frac{2n}{1+n}$.