Question -1. (*10 points*) Let f be the function given by

$$f(x) = \begin{cases} \frac{1}{4}a^2x^2 + 3, & \text{if} \quad x \leq 2, \\ ax^2, & \text{if} \quad x > 2. \end{cases}$$

Find the values of a for which f is continuous at $x = 2$. (Hint: Evaluate the two one-sided limits $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 2^-} f(x)$ to determine its continuity.)

Question -2. (*10 points*) Find $\lim\limits_{x\to 1} \dfrac{x \ln x}{e^x - e}$, if it exists.

Question -3. (*15 points*) Evaluate the integral $\displaystyle\int_{-2}^{2} \sqrt{4 - x^2}\, dx$.

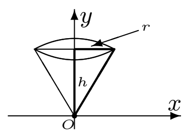
Question -4. (*10 points*) Evaluate the integral $\displaystyle\int \sin^3(x)\, dx$.

Question -5. (*15 points*) Evaluate the improper integral

$$\int_1^\infty xe^{-x}\, dx.$$

Question -6. (*15 points*) Find the exact area of the region bound by the y-axis and the curve represented by the equation $x = 6y - y^2$. (Hint: equation $x = 6y - y^2$ represents a parabola that intersects the y-axis at points $(0, 0)$ and $(0, 6)$).

Question -7. (*15 points*) Let C be a cone with height h and base radius r. Show that the volume of C is $\frac{1}{3}\pi r^2 h$. (Hint: View C as a solid of revolution obtained from some triangle in the xy-plane.)



Question -8. (*10 points*) Find the interval on which the function

$$f(x) = \int_0^x \frac{1}{1 + t + t^2}\, dt$$

is concave upward (Hint: use the fundamental theorem of calculus).