

Question -1. (15 points) Let $f(x) = \cos(x)$. Derive a formula for the n -th derivative, $\frac{d^n}{dx^n} f(x)$ of the function. Explain how this derivative depends on n (even or odd).

Question -2. (15 points) For the given function

$$g(x) = \begin{cases} x \ln(x) & \text{if } x > 0, \\ 0 & \text{if } x = 0, \end{cases}$$

- (a) Evaluate $\lim_{x \rightarrow 0^+} g(x)$.
 (b) Is this function continuous at $x = 0$? Justify your answer.

Question -3. (15 points) Let $F(x) = \sin(x) \cos(x)$, $0 \leq x \leq 2\pi$. Find all critical numbers of this function, and determine whether each represents a local maximum or local minimum.

Question -4. (20 points) (a) Evaluate

$$\lim_{m \rightarrow +\infty} \left(\lim_{n \rightarrow +\infty} \frac{m}{m+n} \right) - \lim_{n \rightarrow +\infty} \left(\lim_{m \rightarrow +\infty} \frac{m}{m+n} \right)$$

- (b) Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin(mx)}{nx}$$

Question -5. (15 points) Find the point on the parabola $y = 9 - x^2$ that is closest to the point $(3, 9)$.

Question -6. (20 points) Find $\frac{dy}{dx}$ at point $(\frac{\pi}{4}, \frac{\pi}{4})$ where x and y satisfy the relation $\sin^2(y) + \cos^2(x) = 1$.