Question -1. (*15 points*) Determine numbers a and b so that $f(x) = \frac{ax}{x^2+b^2}$ has a local maximum at $x = 3$ and that $f'(0) = \frac{1}{3}$.

Question -2. (*15 points*) The gravitational force exerted by Earth on a unit mass at a distance r from the center of the planet is

$$F(r) = \begin{cases} \frac{GMr}{R^3} & if \quad r < R \\ \\ \frac{GM}{r^2} & if \quad r \geq R, \end{cases}$$

where M is the mass of Earth, R is its radius, and G is the gravitational constant. Is F a continuous function of r?

Question -3. (*15 points*) Let $f(x) = x - \sin(x)$, $0 \leq x \leq 2\pi$. On what interval is $f(x)$ concave upward? Justify your answer mathematically.

Question -4. (*20 points*) (a) Evaluate

$$\lim_{x \to 0} \frac{\cos(mx) - \cos(nx)}{x^2}.$$

(b) Evaluate

$$\lim_{x \to 0^+} \sin(x) \ln(x)$$

Question -5. (*15 points*) A rectangle is expanding in such a way that its length is always twice its width. If the perimeter of the rectangle is increasing at a rate of 3 in./min., find the rate of change of area of the rectangle when the area is 24 in.^{2}.

Question -6. (*20 points*) Find $\frac{d^2y}{dx^2}$ at point $(\frac{\pi}{4}, \frac{\pi}{4})$ where x and y satisfy the relation $\sin^2(x) + \cos^2(y) = 1$.