$\mathcal{Q}$uestion -1. (*10 points*) Let f be the function given by

$$f(x) = \begin{cases} \frac{1}{4}a^2x^2 + 3, & \text{if} \quad x \leq 2, \\ ax^2, & \text{if} \quad x > 2. \end{cases}$$

Find the values of a for which f is continuous at $x = 2$. (Hint: Evaluate the two one-sided limits $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 2^-} f(x)$ to determine its continuity.)

$\mathcal{Q}$uestion -2. (*15 points*) A particle moves along a straight line. Its position (in feet) at time t seconds is $P(t) = te^{-t}$.

(i) When does the particle have zero velocity?

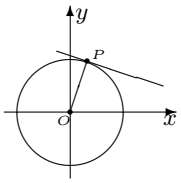
(ii) What is its acceleration at this time?

$\mathcal{Q}$uestion -3. (*20 points*) Find values for a, b, and c such that the function defined by $f(x) = ax^3 + bx^2 + cx$ has a local extremum at $(2, 3)$ and an inflection point at $x = \frac{3}{2}$.
Is this extremum a local maximum or local minimum?

$\mathcal{Q}$uestion -4. (*10 points*) Find $\lim_{x\to 1} \dfrac{x \ln x}{e^x - e}$, if it exists.

$\mathcal{Q}$uestion -5. (*15 points*) A box with an open top is to be constructed from a square piece of cardboard 1 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

$\mathcal{Q}$uestion -6. (*15 points*) Let P be a point on the unit circle $x^2 + y^2 = 1$ and let $O = (0, 0)$. Show that the tangent line to P is perpendicular to $\overline{OP}$. (To keep it simple, you may assume that P is in the first quadrant of the xy-plane.)



$\mathcal{Q}$uestion -7. (*15 points*) Find the exact value of the limit ($c \neq 0$ is a constant):

$$\lim_{x\to 0} \frac{cx^2 + \tan^2(x)}{\sin^2(cx)}$$